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Mathematical Discussions with Linguistically Diverse Students

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Mathematical Discussions with Linguistically Diverse Students

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By

Cecily Stevens

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This action research project, written under the direction of the candidate's master's project advisory committee and approved by members of the committee, has been presented to and accepted by the faculty of the Kalmanovitz School of Education, in partial fulfillment of the requirements for the Master of Arts in Teacher Leadership in Social Justice degree.

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Abstract

Mathematical Discussions with Linguistically Diverse Students
By
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Master of Arts in Teaching Leadership
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Mathematical discussions facilitate linguistically diverse students in achieving deeper understanding of mathematical concepts. Seventh grade students participated in mathematical discussions once a week for twelve weeks, including both large group and small group discussions. Analysis of standardized test scores, summative assessments, student work samples, and audio recordings suggested that mathematical discussions are effective in deepening student's mathematical understand but were especially effective for Standard English Learners. English Language Learners and English Only Learners showed growth during the study but were outpaced by their Standard English Learner peers. Larger community implications are discussed.

Dedication

To my family, who made it possible for me to meet this goal and who never doubted I could do it.

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Chapter I

Introduction

In his first book on mastery learning, Benjamin Bloom asserts that all learners are capable of achieving academic success when provided with quality and attentive education. He writes, “[t]he normal curve is a distribution most appropriate to chance and random activity. Education is a purposeful activity, and we seek to have students learn what we would teach. Therefore, if we are effective, the distribution of grades will be anything but a normal curve. In fact, a normal curve is evidence of our failure to teach” (Bloom, 1968, p.2). Benjamin Bloom’s strong statement holds teachers to a bold standard. In his work, Bloom asserts that memorization is a minimal form of understanding and that its contrary, *creation*, is the deepest form of understanding. In education, this means that students who memorize content may be able to regurgitate information for an upcoming test but will quickly forget everything they have learned thereafter. The Common Core State Standards, in an effort to better prepare students to be college and career ready, focus heavily on deeper understanding pursuing “conceptual understanding, procedural skills and fluency, and application with equal intensity” (National Governors Association Center for Best Practices, 2010). Additionally, math content standards are accompanied by mathematical practices that emphasize the importance that communicating mathematically is much greater than communicating *what* the answer is, but equally important is *how* one arrives at the answer.

This shift to a more balanced focus of deeper understanding aligns with another concerning educational disparity: English Language Learners (ELLs) are being underserved in the mathematical classroom (Howard, 2020). The strong relationship between English language proficiency and math achievement is well-researched (Chen and Chalhoub-Deville, 2016, p.578), and with the shift to Common Core State Standards, students are now explicitly expected to explain the way they think about mathematical processes and be able to communicate their reasoning when solving problems (Wagganer, 2015, p.250).

In the 2020-2021 school year, California served 1,062,290 ELLs (California Department of Education, 2021). Additionally, 257,651 students were classified as Initial Fluent English Proficient and 1,053,625 Reclassified Fluent English Proficient with 84,211 yet to be classified (California Department of Education, 2021). This means that of the approximately 6 million students currently enrolled in California public schools, about 40% of students are working at various levels to master the dominant language of schools while simultaneously working to retain content material in their various subjects. This language barrier is therefore also an achievement barrier, and unfortunately, the research shows that the "achievement gaps in text-level skills between ELLs and native English speakers are large and persistent" (Zhang et al., 2016, p.184).

According to the California Assessment of Student Performance and Progress and English Language Proficiency Assessment for California, the language barrier experienced by many ELLs, along with socio-economic, racial, and other influences, likely contribute to the data from California students in the 2018-19 school year[1], which states that 12.58% of ELLs met or exceeded the standards for Math compared to 44.37% for students who only spoke English and 30.22% for Ever English Learners (current plus former English learners) (2021). This is glaringly problematic for many reasons, but a main one is that "mathematics is often viewed as

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[1] Due to the COVID-19 pandemic, state testing in California was not completed in the 2019-20 school year and a minimal number of schools completed testing in the 2020-21 school year. Therefore, data from the 2018-2019 school year was cited in an effort to show data unaffected by the global pandemic, distance learning, and hybrid learning.

the gatekeeping subject for postsecondary educational access" (Howard, 2020, p.16,18). Therefore, over 85% of ELL students (compared to 55% of English only students) in California face barriers to higher education on their Math proficiency alone.

So, to reach the poignant goal set forth by Benjamin Bloom to facilitate high achievement for all students, the goal of this action research project (ARP) was to help deepen my middle school students' mathematical understanding, especially my ELLs.

Statement of the Problem

English Language Learners are most often identified through a home language survey and/or an English Language proficiency screening assessment. Although identification of ELLs in a timely manner is a federal requirement, no standardized questionnaire, assessment, or guidelines exist (Lopez et al., 2016). Therefore, states and districts vary widely in their ELL identification process (Lopez et al., 2016). In part to fill this void, the California English Language Development Standards were adopted by the California State Board of Education in November 2012 to "maintains California's commitment to providing ELLs with a high-quality program that will enable them to attain proficiency in English" (2014, p.ii). These standards categorize ELLs in three stages: emerging, expanding, and bridging. These stages are recognized as a continuum. Emerging ELLs can utilize and understand basic English words and phrases but may not yet have the fluency to express themselves fully in English. Expanding ELLs are able to communicate effectively in spoken and written English but may be challenged by more complex and technical language. Bridging ELLs are ready to engage in multifaceted and academically demanding language with some support from a teacher. Students who reach this level of proficiency are then reclassified as English proficient but remain designated as Ever English Learners. These standards assume that a student who is classified as an ELL has the resource of one or more other languages besides English and encourages teachers to be aware and mindful of these resources a student may already possess when working with that student to expand their English proficiency.

Taking a broadened perspective of these ELL categories, the Academic English Mastery Program (AEMP) implemented by the Los Angeles Unified School District offers an alternative viewpoint and expanded approach to classifying students' English proficiency (AEMP, 2021). The English language has countless dialects both locally and internationally, including African American English, Mexican American or Chicano English, Hawaiian Pidgin English, and American Indian English (Howard, 2020, p.76). Nonetheless, Eurocentric Academic English is held as the gold standard in higher education and the similarly skewed Standard English is the expectation of the English-speaking business market. The AEMP recognized that many of its students did not speak Standard English at home and struggled with Academic English at school. These students would not traditionally be classified as ELLs as they spoke only English at home, but the Los Angeles Unified School District recognized the need to support these underserved students, who were being excluded from further academic and career opportunities, because their mastery was in an English dialect other than Standard English. So, the AEMP classified these students as Standard English Learners (SEL) to more equitably serve students who would not be classified as ELLs, but who still needed to receive support to reach their full academic potential and to address the achievement gap.

At my own school site, the majority of my students identify as Filipino or mixed Filipino[2] and about a third of students speak another language some, most, or all of the time at home.

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[2] After surveying my 26 students, the majority of students who are of Filipino descent noted that they identify as 'Filipino'.

Comparatively, according to the California School Dashboard and California Data Quest, Filipino American students represent approximately 2.4% of all California students and approximately 1.2% of all California ELLs identified as speaking Filipino (Tagalog) or Ilocano (2021). According to the California Assessment of Student Performance and Progress and English Language Proficiency Assessment for California, in the 2018-19 school year, 59.52% of Filipino students in the state of California met or exceeded the standard for Math and approximately 71.57% for English Language Arts (ELA) (2021). Interestingly, these numbers are reversed for my own students. Approximately 75% of my students are proficient in Math and approximately 45% in ELA. According to the same report, these data are also reverse of the overall California student for the 2018-19 school year that showed that 39.73% of students were proficient in Math and 51.10% in ELA (2021).

Filipino American students, unlike many immigrant populations in California, come with a historical background of English. Because of American colonization, English has been spoken in the Philippines, including use in their education system, for several generations (Halagao, 2002, p.44-45). Also, due to their colonization by the United States, some Filipino Americans may have a colonial mindset and therefore a desire to be as ‘American’ as possible, including speaking English (Halagao, 2002; Nadal, 2008; Mendoza and Parba, 2019). I have observed that many of my students are second or third generation Filipino Americans and while most are proud of their Filipino heritage, some wish only to be seen as American. I have also observed that approximately a third of my students speak another language at least some of the time at home. The students predominately name this home language as Tagalog[3], one of two of the state

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[3] Tagalog is one of several native languages spoken in the Philippines. Parba argues that Filipino would be a more inclusive and fluid name for this language but recognizes that many Filipino Americans prefer to use the term Tagalog (Mendoza and Parba, 2019).

recognized language of the Philippines, the other being English (Mendoza and Parba, 2019). As my school is a private institution, it is not mandatory that we collect home language data. Therefore, we have no formal data in this area and this information has been gathered informally through teacher-student observation and interaction. Additionally, I have observed that parents and grandparents of the Filipino American students in my community may not show full fluency of the English language or may have heavy accents when speaking English. However, many of our Filipino parents work in the medical field as medical professionals, so their English language fluency and mathematics fluency should not be underestimated.

Within this context, I believe that while some of my students would be identified as ELLs, others would more likely be identified as SELs, as evidenced by the differential in their ELA and Math proficiency scores and their familial language dynamics. However, regardless of classification, I have observed that many of my students struggle with Academic English in my Math classes. This is most evident in their spoken and written mathematical responses. My ARP aims to bring to focus the language complexities of my students and to use this understanding as a driving factor of my instruction.

Purpose of the Research

Even before the Common Core Standards, educators were working with ELLs to explicitly acquire vocabulary specific to mathematical topics and their academic usage (Moschkovich, 2012, p.304). But educational researchers point out that there is much more than vocabulary in the language of mathematics (Moschkovich, 2012; Sigley and Wilkinson, 2015; Zahner et al., 2012). The language of mathematics is “not to mean a list of vocabulary words with precise meanings but the communicative competence necessary and sufficient for competent participation in mathematical discourse practices” (Moschkovich, 2012, p.304). Dr.

Moschkovich, a professor at the University of Santa Cruz, goes on to write that English Language Learners can and should participate in mathematical discussions regardless of their English proficiency, again citing the Common Core Standards and its demanding and relentless focus on the student’s ability to answer the question adequately and articulately: why? (Moschkovich, 2012, p.305). In a research study that Dr. Moschkovich co-authored, teachers were more successful with ELLs when they engaged in mathematical discussions and allowed students the time and space to reason and justify their work out loud (Zahner et al., 2012). Their study shows findings that contradict the assumption that English Language Learners need to be spoon-fed mathematical language and, in this instance, actually negatively impacted the achievement scores compared to students who were given more agency in their math classes. In the study, students who were seen as sources of knowledge and valued contributors to class discussion showed improved mathematical scores, while students who received more direct instruction from the teacher and had little to no opportunities to contribute in class showed lower scores than their peers.

In my own classroom, I have witnessed the struggle students have when answering Common Core questions that require justification and reasoning, especially among my ELLs and SELs. Sigley and Wilkinson (2015) discuss the connection of student understanding and mastery of developmentally appropriate mathematical discussion, or what they term the mathematical register. If a student fully understands a topic, they will be more able to make use of the mathematical register, meaning that deeper understanding and a robust explanation are notably linked (Sigley and Wilkinson, 2015). My middle school students are highly engaged by any chance to socialize with their peers. They are more than happy to discuss an academic topic presented by the teacher, even if they sometimes deviate from the focus of the lesson. However, I am often pleasantly surprised by the depth and insight a middle school student can reach when given the opportunity. With a little structure and guidance, I believe mathematical discussion would be highly engaging and beneficial for my students and allow them to bring to the lesson new insights that would be lost if only direct instruction was implemented. I believe these discussions will also afford students the agency and independence that most middle school students are beginning to crave. My hope is that this technique elevates and engages my students, especially my ELLs and SELs.

These small group mathematical discussions were meant to initially be conducted using everyday language, allowing students of all English language proficiency levels to contribute in any way they felt comfortable. Each group focused their discussion around the same problem or set of problems and students were expected to write down their thinking. The teacher circulated during these discussions and offered redirection and academic language support. The students were then asked to share out their answers to the class followed by a large group discussion to synthesize and clarify the answer to the problem(s). Deeper mathematical understanding should be evident in the later mathematical discussions and writing, as evidenced by an increased use of mathematical language. This deeper understanding should also show through in benchmark standardized test scores, as well as summative assessments.

Action Research Question

The action research question for this study was: *How do mathematical discussions impact middle school students' deeper understanding of mathematical concepts, particularly for English Language Learners?* I hoped to see students' increased use of the mathematical language both spoken and written, and I hoped to see their achievement scores on standardized and summative assessments rise as a result of mathematical discussions.

Limitations

This research project presented several limitations that may impact the outcomes of this study. Due to the nature of this project, limitations include limited time, a small sample size of students, and a skewed population. This study took place over the course of twelve weeks, limiting the information that could be gathered and growth that could be measured. Secondly, the sample population was unique to my school site and my particular students. Some of my ethnic populations were very small, being made up of only one or two students. This significantly skewed the percentages of the data for my Black, White, Middle Eastern, and non-Filipino Asian populations. Finally, students who were already identified as needing extra support were not included in this study, as they were provided more individualized instruction by another teacher in a different classroom space. Therefore, the students in this study were all identified as general or high-achieving mathematics students. These constraints limited the viability of generalizations from the findings of this study.

Additionally, this study took place during the COVID-19 pandemic. Students were also required to stay home if they had any cold or flu symptoms and to obtain a negative test result before returning to school, which sharply increased the number and duration of student absences and made consistent collection of qualitative data very challenging. Learning loss was also evident from the previous year of distance learning. Although all standards were covered during distance learning, students showed lower retention and mastery of these foundational standards. When these standards were revisited during the current school year, re-teaching was often necessary before grade level standards could be taught.

Positionality of the Researcher

I am a white cisgender woman, who researched racially and ethnic diverse learners in a suburban school in Northern California. Over the course of my seven years of teaching at this school site, I observed that many of my students struggle with language-related barriers in their mathematics work, especially when measured by Common Core expectations of robust mathematical explanations. As the lead math teacher in the middle school, I wanted to better reach these students to ensure quality and equitable education for all of my students. However, my life experience is one of abundant privilege. Therefore, I must be ever mindful of my power and privilege and do my best to avoid its influence in my teaching and interactions with my students.

Furthermore, I hope to be mindful of my biases. Because I have taught at this school site for several years, I have made connections with many of the families who may have attended our school from infancy through eighth grade with multiple children. Because of these relationships and previous interactions, my objectivity is impaired. To minimize this, I will use empirical evidence in addition to my observations and triangulate my data.

Additionally, based on my previous teaching experience with middle school students, I assumed that my students would be willing to participate in small group discussions and that they would in fact prefer small group work over whole class instructions. I assume students will contribute to the mathematical discussions with little or no prompting based on the culture and expectations of my classroom and my school.

Definition of Terms

English Language Learner (ELL)

Learners who are working to gain full fluency of the English language and who speak a language other than English at home some, most, or all of the time

Ever English Learners (Ever EL)

All learners who are currently classified as English Language Learners and those who have been re-designated as Reclassified Fluent English Proficient ("*California Assessment of Student Performance and Progress and English Language Proficiency Assessment for California"*, 2021).

English Only Learners (EO)

Learners who are exposed to only the English language at home by adults whose first language is also English

Filipino American

A person whose intersecting identify includes both Filipino and American cultural influences

Mathematical Register/Language

The full scope of spoken and written vocabulary, syntax, and norms associated with academic mathematics (Moschkovich, 2007; Moschkovich, 2012; Sigley and Wilkinson, 2015.).

Standard English Learner (SEL)

Learners who are working to gain full fluency of the Standard English Language (AEMP, 2021) and who are mostly exposed to the English language at home from adults whose first language is not English

Zone of Proximal Development (ZDP)

The stage of learning when a person has the potential to learn new concepts with the assistance of a more knowledgeable person (Vygotsky, 1986)

Implications

The purpose of this research was to positively impact the deeper understanding of mathematical concepts for diverse middle school students through mathematical discussions. This study addressed the math opportunity gap present between ELLs and their EO counterparts statewide and in my own classroom. Mathematical discussions are a powerful tool to engage, assess, and deepen understanding for students who may not have full fluency in English (Moschkovich, 2012). Studies show that mathematical discussions increased student engagement and mathematical understanding (Moschkovich, 2012; Sigley and Wilkinson, 2015; Zahner et al., 2012). This practice is both powerful and equitable as ELLs often face barriers to higher education and thus the accreditation to engage in higher paying work (Howard, 2020).

If the results of this study are positive, I may look to other grades in my school to examine achievement gaps and work to help other teachers implement mathematical discussions more regularly in their lessons. On a small scale, the findings of this inquiry can be shared among local schools in one of the most diverse counties in the nation. On a grander scale, this work supported current research for the need to engage diverse students in new ways to ensure equitable practices.

Chapter II

Literature Review

The purpose of this action research project was to help deepen my middle school students' mathematical understanding through mathematical discussions, with a focus on my English Language Learners (ELLs) and Standard English Learners (SELs). I believed mathematical discussion would be highly engaging and beneficial for my students and allow them to bring to the lesson new insights that would be lost if only direct instruction was implemented. I believed these discussions would also afford students the agency and independence that most middle school students are beginning to crave. My hope was that this technique elevated and engaged my students, especially my ELLs and SELs.

Even before the implementation of the Common Core State Standards, educators were working with English Language Learners to explicitly acquire vocabulary specific to mathematical topics and their academic usage (Moschkovich, 2012, p. 304). But educational researchers point out that there is much more than vocabulary in the language of mathematics (Moschkovich, 2012; Sigley and Wilkinson, 2015; Zahner et al., 2012). Dr. Moschkovich wrote that ELLs can and should participate in mathematical discussions regardless of their English proficiency, again citing the Common Core Standards and its demanding and relentless focus on the student's ability to answer the question adequately and articulately: why? (Moschkovich, 2012, p. 305).

Sigley and Wilkinson also discussed the connection of student understanding and mastery of developmentally appropriate mathematical discussion, or what they termed the mathematical register (2015). If a student fully understands a topic, they will be more able to make use of the mathematical register, meaning that deeper understanding and a robust explanation are notably linked (Sigley and Wilkinson, 2015). In other words, producing spoken and written mathematical explanations appropriately using the mathematical register was a measure of deeper mathematical understanding.

Therefore, the action research question that guided this study was: *How do mathematical discussions impact middle school students' deeper understanding of mathematical concepts, particularly for English Language Learners?*

Overview of the Literature Review

This literature review begins with the theoretical rationale which focuses on the distinguished educational theorists, Lev Vygotsky (1978) and Stephen Krashen and Tracy Terrell (1998), who established the importance for students to gain new knowledge within proximity of prior knowledge through the guidance of a more knowledgeable person. Then, the related research reviews four sections: language and math proficiency connection, mathematical register and academic literacy in mathematics, mathematical discourse and ELLs, and mathematical communication as a measure of deeper understanding. The relevant research articles reviewed in this study were collected through thorough research investigation from the following academic databases: SAGE, ERIC, Journal of Urban Mathematics Education, JSTOR, Springer, Journal of Latinos and Education, Journal of Mathematical Behavior, and Elsevier. The research for this literature review was guided using the following key terms: mathematical registers, mathematical discourse, mathematical discussions, mathematical academic literacy, math proficiency, language proficiency, culturally responsive mathematics, English Language Learners, Filipino American learners, Common Core, mathematical writing, writing to learn, and think-alouds.

Theoretical Rationale

The two theories that were used to frame this action research project were Lev Vygotsky's zone of proximal development (ZPD) (1978) and Stephen Krashen and Tracy Terrell's (1998) natural approach to language acquisition. Lev Vygotsky's zone of proximal development was a guiding theory in that it recognizes the child's potential to learn new concepts with the assistance of a more knowledgeable person even if they could not acquire these concepts on their own. A mindful teacher uses this intermediate learning zone to introduce new ideas with appropriate support to push students further in their learning. Krashen and Terrell recognize this concept in their own theory and its implications for language acquisition. The *natural theory* asserts that language is acquired most effectively through natural dialogue as opposed to structured language lessons. According to Krashen and Terrell's research, caregivers and teachers naturally speak to children at a level they can understand because their primary goal is to be understood and to communicate with the child(ren). They also naturally stretch children just beyond their current language acquisition by using context clues to convey the meaning of new language or by limiting topics to the present time and place to avoid confusion. In a very ordinary way, this language acquisition is happening in the ZDP.

In this literature review, Lev Vygotsky's zone of proximal development underpinned the overall design of this project, therefore, it is discussed first. Krashen and Terrell's natural approach follows as it cemented the ZDP specifically in language acquisition. These theories contributed to the action research project and helped to shape the design and implementation of the project.

Zone of Proximal Development

The research design of this thesis was greatly informed by Lev Vygotsky's zone of proximal development theory, which Vygotsky defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). In practice, this means that a first grader will not likely be completing calculus problems but could easily be taught basic subtraction by a teacher following the child's understanding of basic addition. This interaction allows the child to achieve a new skill that they would not have been able to complete on their own. Vygotsky noted the importance that these interactions are intended to be social, allowing the teacher and the learner to engage in collaborative dialogue. Through these interactions, a child can learn by example and through targeted verbal instructions. It is also important that the teacher or mentor provide support for the child in the ZDP. This provided support came to be termed scaffolding (Wood, Bruner, and Ross, 1976). Scaffolding is specific to the learner and is fluid through the ZDP. As a child approaches mastery of a new skill, less and less scaffolding is needed. Scaffolding may come in the form of manipulatives, modeling, or hints. According to Vygotsky, the child internalizes and integrates the information and modeling presented by the teacher to then use in their own way when independently performing a new skill (Vygotsky, 1986, p. 19).

The Natural Approach to Language Acquisition

Another theory that provides guidance for educators in moving students to the next level of achievement is Stephen Krashen and Tracy Terrell's (1998) natural approach to language acquisition. Krashen and Tracy focused their work on acquiring language not through grammar drills or error correction, but by "understanding input that is a little beyond [the] current level of (acquired) competence” in a low stress environment (Krashen and Terrell, 1998, p. 32). According to Krashen and Terrell’s research, caregivers and teachers naturally do this, because their purpose is to be understood and to communicate with the child. They naturally speak to children in their linguistic ZDP, so the child will be able to comprehend the communication. Natural communication is the corner stone of Krashen and Tracy’s theory. They stated that when “we ‘just talk’ to our students…we may be giving them the best possible language lesson since we will be supplying input for acquisition” (Krashen and Terrell, 1998, p. 35). In this way, the natural approach made use of the ZDP of language learners in naturally occurring classroom conversation. Krashen and Tracy went on to contend that this approach supersedes standard grammar exercises, because it could further students at different language levels simultaneously and it was almost always more interesting to students.

Krashen and Tracy outlined ideal situations for the natural approach to occur in the classroom, one of which was problem-solving activities. In such a situation, students are engaged in a collaborative effort to solve a problem in the target language without language acquisition being the focus. However, teacher input and peer input become the source for language acquisition (Krashen and Terrell, 1998, p. 108). In this instance, the learner was situated in the ZDP with scaffolding provided both by the teacher and their peers.

Review of Related Research

The review of the associated literature was organized into four sections: language and math proficiency connection, mathematical register and academic literacy in mathematics, mathematical discourse and ELLs, and mathematical communication as a measure of deeper understanding. This literature emphasized the relationship between English language proficiency and mathematical proficiency in English-speaking, American schools. The more recent literature also redefined mathematical vocabulary to include academic syntax and mathematical language norms. Furthermore, this research review focused on developing students' ability to discuss mathematical topics in both the vernacular and in traditionally academic ways. Finally, this review of the related research provided evidence that mathematical communication is a measure of deeper mathematical understanding.

Language and Math Proficiency Connection

Though often seen as distinctly separate, English language arts (ELA) and mathematics school courses and their associated skill sets have a strong interplay with each other. In this section, the following two studies linked language proficiency and math achievement and examined why mathematical language can be a barrier for some students.

In a study conducted by Chen and Chalhoub-Deville (2016), roughly 21,409 students were tracked from kindergarten to eighth grade. The data was sourced from the Early Childhood Longitudinal Study, Kindergarten Class of 1998–99. The U.S. Department of Education, Institute of Education Sciences, and National Center for Education Statistics funded this project. This research was designed to analyze the long-term relationship between language and math achievement accounting for socioeconomic status, gender, and ethnicity background influences. The study found that the "READING [score] by itself explains 44–54% of the variance in MATH scores at each grade" (Chen and Chalhoub-Deville, 2016, p. 583).

This study was limited by the incomplete usage of the READING score to measure language proficiency, despite the multitude of factors that influence language proficiency. The researchers also pointed out the limitation that the data was not representative of the current U.S. population of students, but they believed that the results would correspond if compared.

The results of this study matched the current understanding that American math achievement scores are heavily impacted by English language fluency. However, while this study did find that the overall categorization of ELLs had a strong statistical correlation with math achievement scores, the specific ELL status of a student, such as immerging or advanced, did not.

In another article, Mary Schleppegrell (2007) brought together several resources to discuss the linguistic challenges of mathematical learning. The field of mathematics has its own vocabulary as does any discipline. However, mathematics also uses natural words in new and very specific mathematical ways, and mathematics has an adjacent language of symbols and visual representations. Schleppegrell also highlighted the grammar construction found in mathematical language, which includes dense noun phrases, being and having verbs, conjunctions with technical meaning, and implicit logical relationships (2007). These systemic and grammatical complexities make mathematics challenging for all learners to access, but especially challenge ELLs and SELs. Schleppegrell suggested that teachers should be aware and knowledgeable of the heavy influence of mathematical language on students' ability to gain high level math knowledge, as well as express their own thinking about mathematics. She advised that teachers provide opportunities for students to use the mathematical language in both its spoken and written forms. The teacher and the textbook could be valuable resources for students in gaining understanding of mathematical language, but self or peer exploration and practice were important, too. Schleppegrell recommended whole class exploration of a complex problem with guidance from the teacher as a highly effective method for students to grapple with a mathematical concept but scaffolding mathematical language may still be needed.

In my own classroom, I have observed that students struggled to explain their thinking in both written and oral formats. They understood the mathematical concepts enough to solve number-only problems accurately but struggled in their attempts to explain their work using mathematical language, or what some researchers term, the mathematical register.

Mathematical Register and Academic Literacy in Mathematics

In this section, the history and a modern interpretation of the mathematical register was examined from both a linguistic and an educational perspective.

In 1978, linguist Michael Halliday wrote about the mathematical register as language specific to mathematical concepts (Halliday, 1978). A decade later, David Pimm situated Halliday's work within the context of the English language and in the English-speaking classroom (1987). Pimm described the mathematical register as complex, including not only subject-specific vocabulary, but also phrasing, syntax, and the use of vernacular words in different and exclusive ways (1987). Later, Candi Morgan argued that the mathematical register could not be quantified, because it so heavily depended on intention, context, and those who invoke its use (1998). However, a more inclusive definition might be that the "mathematics register is a complex construct that includes styles of meaning, modes of argument, and mathematical practices and has several dimensions such as the concepts involved, how mathematical discourse positions students, and how mathematics texts are organized" (Hunter, 2017, p. 67).

Dr. Judit Moschkovich, who has been researching mathematics education with ELLs for nearly three decades, adamantly refuted the idea that the acquisition of mathematical vocabulary should be the focus of linguistically diverse classrooms. Dr. Moschkovich (2015) asserted that language is situated in sociocultural norms. Therefore, the mathematical language, regardless of the actual language being spoken, was far more than word definitions. Words can have multiple or changing meanings given context and setting. As Moschovich notes, “learners negotiate situated meanings for words and phrases that are grounded in the local sociocultural setting and coordinated with ways of viewing inscriptions” (Moschkovich, 2015). Built on the work of James Gee and Brian Street (Gee, 1999; Street, 2005), Dr. Moschkovich defined mathematical academic literacy as including vernacular language and multimodal communication, such as “images, equations, symbols, sounds, gestures, graphs, and artifacts” (Moschkovich, 2015).

Therefore, students must navigate complex mathematical language and negotiate the differences between vernacular and mathematical meanings of words, phrases, and syntax. This can be challenging for all students, but is especially challenging for ELLs, who are navigating a double codeswitch from a home language to Standard English to mathematical language. Dr. Moschkovich suggested that an empowering way to navigate this challenging work is to engage students in talking about mathematics.

Mathematical Discourse and ELLs

Dr. Judit Moschkovich is a strong proponent of the practice of mathematical discourse (Moschkovich, 2007). As middle and high school students are often highly engaged in class activities that are socially oriented, mathematical discussions are an ideal classroom practice for this age group. This practice also holds space for students to be sources of knowledge, rather than passive receivers of curated information. The following studies highlighted the powerful effect mathematical discussions had for all students, but especially for ELLs.

Dr. Moschkovich’s definition of mathematical discourse places heavy emphasis on students making meaning through discussion. The teacher, the textbook, and student experience are all held as funds of knowledge, without one outweighing another. In mathematical discourses, the teacher is a facilitator who buffers and redirects student conversation rather than the teacher being an all-knowing guide moving students to a particular conclusion. Students construct their own meaning and work through their understanding of concepts collectively with their peers. Therefore, mathematical discussions are safe places for students to explore ideas and come to their own conclusions with the support of the teacher.

In her 2007 study, Dr. Moschkovich observed a teacher who guided a mathematical discussion toward a textbook definition answer and did not leave room for student interpretation. She argued that this approach negates student competency. The book and the student can both be correct, simply stated differently. The aim of Moschovich's inquiry was to investigate the features and characteristic of mathematical discourses compared to everyday language. The findings of the research pointed out that everyday language can be incredibly useful for students to explore mathematical ideas and that restricting students to academic mathematical language impeded students from sharing their full knowledge of the topic, based on their current language acquisition levels. Dr. Moschkovich argued that mathematical discourse should include vernacular and informal language. Moreover, utterances, gestures, and the nonverbal references to visual aids (such as graphs) should be seen as valid forms of communicating ideas and as important ways of moving the conversation forward. All students could gain understanding of mathematical skills, complex mathematical concepts, and socio-cultural mathematical norms through negotiating and grappling with real mathematical problems through social interactions with peers and with the teacher.

In another study, Song and Coppersmith, examined the importance of using discussion-based math instruction as a significant tool to help ELLs gain not only vocabulary, but also math competency and math specific language skills (2020). The latter is a key part of the Common

Core State Standards, which call for all students to be able to reason and justify their mathematical thinking. This study looked at three teachers teaching the same lesson using the same curriculum. The study measured the achievement impact of teacher mode of instruction and how the teacher structured the mathematical discourse to promote students' conceptual understanding with students who are multilingual. About 800 linguistically diverse, Latinx students participated in the study in the Rio Grande Valley region of Texas, which borders Mexico. "Over 95% of the students in each class were Latino/a, and in each of the three schools, over 85% of the students were eligible for free or reduced-price lunch" (Song and Coppersmith, 2020). The researchers video recorded the classes and transcribed the in-class conversations for analysis. The study found that the more successful teachers moved more slowly through the lesson, even though they covered less content. Space was given for students to talk and reason with and without teacher guidance. The more successful teachers also introduced new vocabulary when needed and when it naturally occurred in the discussion, as well as, responded to student errors by repeating wrong responses and asking for more answers or asking guiding questions to promote the continuation of the discussion and deeper thinking. This type of instruction also gave more agency to students and did not hold the teacher to be the sole source of information in the classroom. The study dispelled common misunderstandings of working with ELLs and justified with corresponding achievement scores that ELLs can and should participate in discussion-based mathematical learning, regardless of their language levels.

Collectively, these studies conducted by Moschkovich (2007) and Song and Coppersmith (2020) emphasized the importance of mathematical discourse in the classroom, especially for ELLs. Through discourse, students were better able to gain new mathematical understanding and still hold agency in their own learning. This culturally responsive mode of instruction promoted quality learning for all students.

Mathematical Communication as a Measure of Deeper Understanding

In a study conducted with 62 middle school math students, researchers first asked students to evaluate the importance of writing, reading, and talking when learning math (Liedtke and Sales, 2001). Of the students who were surveyed, about half thought that these less traditional methods of mathematical instruction helped them learn new math skills. However, the framework of curriculum designed by the British Columbia Ministry of Education (Ministry of Education, 1995), the principles and standard preferred by the National Council of Teachers of Mathematics (NCTM, 2000), and the advised key shifts in mathematics education guided by the Common Core State Standards (National Governors Association Center for Best Practices, 2010), all emphasis mathematical communication as a critical methodology to lead students to deeper mathematical understanding. The researchers of this study cited Van de Walle's assertion that "students who are able to understand a procedure conceptually, an important goal of teaching and learning mathematics, can think, talk, and write about [that procedure]" (Van de Walle, 1994, p. 33). The study proceeded to implement regular opportunities for writing and discussion in the focus math classroom. The study noted that of the writing opportunities provided, students were particularly engaged by writing their own story problems, creating their own review materials, and designing stories to match graphs. By the end of the study, approximately 80% of students thought that mathematical writing and discussions helped them learn in their math class.

In another study conducted in 2015, Dr. Christie Lynn Martin, of the University of South Carolina, focused on mathematical writing in a fourth-grade classroom over the span of six weeks. Dr. Martin and the classroom teacher worked together to design and implement a revised version of Writers Workshop (Lucy Calkins) specifically geared toward mathematical writing in conjunction with discussion and conferencing. Dr. Martin noted the well-researched idea of *writing to learn*, stating that writing “enhances students’ ability to reflect, strategize, and communicate” (Martin, 2015, p. 303-304). At the same time, writing was an excellent assessment tool for teachers to measure student understanding, misconceptions, and ways of thinking far more than a number-only problem would be.

The researcher noted that this study was a case study and thus limited by the nature of a small and specific group of participants. However, the study overwhelmingly found that its participants who started out with jumbled and incoherent mathematical writing showed tremendous growth in a short time frame and their writing even “highlighted changes in their thinking and illustrated a movement toward more efficient and sophisticated calculations” (Martin, 2015, p. 312). The teacher also found that the writing provided her with a much more detailed and accurate assessment of student understanding and misconceptions. Dr. Martin concluded that explicitly focusing on mathematical writing together with Writer’s Workshop style conferencing and discussion helped students meet Common Core State Standards expectations “to communicate about mathematical concepts in a clear and coherent manner” (Martin, 2015, p. 311).

These studies conducted by Liedtke and Sales (2001) and Dr. Martin (2015) stressed the multiple benefits of mathematical writing in the classroom. Mathematical writing was both a tool for the student and the teacher. The student was pushed to organize and synthesize their thinking when they express their mathematical understanding through writing. The teacher was then able to access a richer assessment of student learning and misconceptions. Mathematical writing both pushed the student to a higher level of understanding and allowed the teacher to monitor and assist their learning more closely.

Summary

This review of the literature emphasized the influence of Lev Vygotsky, as well as Stephen Krashen and Tracy Terrell and the influence of their ideas on the framing of this project. Vygotsky and his successors examined the importance of scaffolding and the methodology of stretching students to the next step of learning through the zone of proximal development (Wood, Bruner, and Ross, 1976; Vygotsky, 1978). Krashen and Terrell built on this understanding, encouraging teachers of ELLs to apply this same progression when helping students in building new language skills (Krashen and Terrell, 1998). Both theorists underscored how learners should be supported socially as they develop new skills and understandings.

The related research examined the intersecting relationship between language and math, in addition to, the mathematical register/language, mathematical discourse and their benefits for ELLs, and how mathematical communication is a measure of deeper mathematical understanding. This research bought to light many best practices for teachers working with ELLs, but it also emphasized how math teachers can be agents of social justice in the classroom (Moschovich, 2007; Song and Coppersmith, 2020). Through understanding and practice, math teachers used these strategies to engage and encourage students who historically underperform in academic mathematics compared to their peers. At the same time, teachers motivated all students to strive for greater mathematical understanding and they moved all students toward deeper mathematical understanding.

Based on this research, mathematical discussions are a proven mode for students of all language levels to actively participate in robust mathematical learning, as well as, providing students ownership in their own learning. In the next chapter, the specific methods used to achieve the goal of implementing mathematical discussions in my classroom to promote deeper mathematical understanding will be detailed and explained.

Chapter 3

Methods

The implementation of Common Core Math State Standards has placed a notable emphasis on deeper understanding and critical thinking, rather than rote memorization and calculation. Students are expected to be able to justify and support their mathematical choices and thinking, which is often assessed through mathematical writing. However, using mathematical language to express understanding can be difficult for students. The systemic and grammatical complexities of the mathematical language make mathematics challenging for all learners but can be especially challenging for English Language Learners (ELLs) and Standard English Learners (SELs) (Moschivitch, 2012; AEMP, 2021). In my nearly decade of teaching Common Core Math Standards, I have observed the struggle my students have when explaining their thinking, especially at my current school site, where many of my students are ELLs and SELs. This disparity was reflected in the 2018-19 school year data, which states that 12.58% of ELLs met or exceeded the standards for Math compared to 44.37% for students who only spoke English and 30.22% for Ever English Learners (current plus former English learners) (California Department of Education, 2021). Though not formally identified, I noted through informal observation and student-teacher conversation that about a third of the students in this study spoke another language at home some, most, or all of the time and would in a public school setting most likely be classified as ELLs. I also observed that about a third of my students regularly interacted with adults at home whose first language was not English. These students have been identified as SELs in this study. The remaining third mainly interacted with adults whose first language was English and have been identified in this study as English Only learners (EOs).

While students in this study showed strong mathematical skills with all student scoring above the 50th percentile nationally and over half scoring above the 75th percentile, only 75% scored above the 50th percentile nationally in English Language Arts (ELA) and only 25% scored above the 75th percentile. The research showed that the "READING [score] by itself explains 44–54% of the variance in MATH scores at each grade" (Chen and Chalhoub-Deville, 2016, p. 583). This differential was evident not only in my student's standardized test scores, but also in their written mathematical responses on summative assessments. I have observed that many students showed mastery of a skill when an assessment question is purely numerical but struggled to explain their reasoning in a written response.

The research also noted that the intersecting relationship between language and mathematics can significantly influence the student's ability to master the mathematical register, modernly defined as the full scope of spoken and written vocabulary, syntax, and norms associated with academic mathematics (Moschovich, 2012; Sigley and Wilkinson, 2015). Dr. Judit Moschkovich, who has been researching mathematics education with ELLs for nearly three decades, strongly suggests that ELLs can and should participate in mathematical discussions regardless of their English proficiency to gain better command of the mathematical register (Moschkovich, 2012). Because middle and high school students are often highly engaged by socially oriented class activities, I believed mathematical discussions were an appealing and effective mode of mathematical learning. This practice also holds space for students to be sources of knowledge, rather than passive receivers of curated information. Dr. Moschkovich's studies highlight the powerful effect mathematical discussions can have for all students, but especially for ELLs.

The purpose of this action research project was to help deepen my middle school students' mathematical understanding through mathematical discussions, with a focus on my ELLs and SELs. Therefore, the action research question that guided this study was: *How do mathematical discussions impact middle school students' deeper understanding of mathematical concepts, particularly for English Language Learners?*

Setting

The school where this study was conducted was located in a suburban area in Northern California. The school was a private, religious school, which required students to pay tuition or maintain scholarships. This requirement skewed student attendance to those from upper-middle and upper socio-economic families. The approximately 5-acre campus included two middle school buildings, a preschool and infant care building, a church, a church office building, several outdoor spaces, and a main building housing Transitional Kindergarten (TK) through fifth grade, the parish hall, the main school office, extended day care, and three small group learning areas. The school was situated among an older, single-family housing development.

All students TK through eighth grade had access to one-to-one technology with the guidance of one and a half full time technology coordinators. There was an instructional assistant in every class TK through fifth and one who covered all middle school grades. There were also two full time special needs educators, and two additional instructional assistants, who facilitated daily small group pullouts for kindergarten through eighth grade math and ELA. These small groups strived to stay on pace with typical curriculum, but also reviewed topics as needed. There were also four full or part time special subject teachers on staff, who provide instruction in music, physical education, Spanish, and fine arts. There was one full time chef, who prepared lunches for the whole school. There were 45 total employees on payroll, including the infant care, preschool, TK through 8th grade homeroom, instructional aides, teacher specialty teachers, and office staff.

At the time of this study, over 400 students attended the school, of which approximately 85% attended TK through eighth grade. Of these students, approximately 50% identified as boys and 50% identified as girls. Approximately 30% percent identified as White or Caucasian, 40% identified as Asian, 5% identified as Black or African American, and 25% identified as Other Races. Additionally, approximately 20% identified as Hispanic or Latino. Approximately 90% of families identify as the same religious affiliation as the school. The predominant cultural influence at the school is Filipino and many students identify as fully or partially of Filipino decent. However, the above data may be misleading because the categories were very limited. The school had several families who had parents who identified as different races and there was no category for Two or More Races on the school survey. The data might also be misleading as some people of Filipino decent identify as White, Hispanic, or Pacific Islander, rather than Asian.

The school community was heavily influenced by Filipino American culture. Filipino Americans are more than 80% Catholic (Halagao, 2002, p.42). Many of the families who attend the school also attended regular church services. Several Catholic Filipino holidays were attended and supported by the school staff and faculty throughout the year. In addition, many Filipino American parents and extended family members were willing and able to participate in school activities, during or outside of school hours. The Filipino presence and greater community involvement was very strong at my school site. And as Halagao points out, "[m]ost Filipino-American students do well in environments that promote a sense of community" (Halagao, 2002, p.44), a sentiment that fairly could be applied to most any grouping of human peoples.

This school site did not classify students as English Language Learners, however, I informally observed that about a third of the students in this study spoke another language at home some, most, or all of the time and would in a public school setting most likely be classified as ELLs. I also observed that about a third of my students regularly interacted with adults at home whose first language was not English. These students have been identified as SELs in this study. The remaining third mainly interacted with adults whose first language was English and have been identified in this study as EOs. Furthermore, the school qualified for Title 1 and 2 funds, due to approximately 20% of students qualifying for free or reduced lunches. Approximately 5% of students had diagnosed and documented special needs and received accommodations and/or modifications.

Of the 10 homeroom teachers, seven held California teaching credentials and two had area specific certification. Four of these teachers also held or were actively working towards master's degrees in education during the time of this study. Approximately half of the homeroom teachers identified fully or partially as Filipino and the other half identified as White or Caucasian. These teachers predominately identified as women. The instructional assistants, specialty teachers, and special education teachers also predominately identified as White or Caucasian women, but this group did include select individuals who identified as Latinx, Pakistan, or men.

In the fall of 2021, approximately 75% of students first through eighth grade were scoring at or above the 50th percentile nationally in ELA and approximately 90% of students were scoring at or above the 50th percentile nationally in Math as measured by the STAR Renaissance standardized test, which was administered every six weeks during the school year.

It is important to note that this study took place during the COVID-19 pandemic. At the time of the study, the school remained open, but students and teachers were masked at all times. Students were also required to stay home if they had any cold or flu symptoms and to obtain a negative test result before returning to school, which sharply increased the number and duration of student absences. The lasting effects of distance learning from the previous year of instruction also had an impact on student learning and, therefore, the findings of this study.

Demographics of the Classroom

The participants of this study were from my seventh-grade class during the 2021-2022 school year. At this school site, there was only one class per grade level, averaging thirty students per middle school class. Middle school students at this site moved to different classrooms and teachers for each of their subjects throughout the day. All seventh-grade students participated in this study, except for the small group of pullout students, who regularly met with an instructional assistant during our math class period. There were 26 students total in this study who were considered general education or high-achieving learners. About 46% of these students identified as young women and 54% identified as young men. Approximately, 73% of the participants identified as Asian or Pacific Islander, which included Chinese, Japanese, and Vietnamese, but predominately Filipino. Approximately, 20% identified as Two or More Races, and less than 10% identified as Hispanic or Latino, less than 10% identified as Black or African American, and less than 10% identified as White or Caucasian. Daily instruction of all middle school classes was conducted fully in English, excluding Spanish language classes. Students with identified or documented special needs were in the small, pullout Math group, so the students in this study were all considered general education or high-achieving learners. Less than 10% of participants qualified for free or reduced lunches.

Data Collection Strategies

A variety of data were utilized in this study and were gathered from the following sources: STAR Renaissance standardized tests, audio recordings of students' mathematical discussions, summative student assessments, and students' class work (see Appendices 1-4). This variety of qualitative and quantitative data upholds the validity of the results obtained in this study. All identifiers were removed and replaced with pseudonyms for this study.

STAR Renaissance Standardized Tests

STAR Renaissance was a national standardized test that evaluates students' mastery of CCSS and compared their scores both to an internal scaled score as well as other students who took the test recently in the United States across multiple measures. The test was adaptive and adjusted the sequences of questions based on the accuracy of previous questions, so every test was unique to the student each time it was taken. The test asked approximately 35 questions and covered all CCSS domains: ratio and proportional relationships, number systems, expressions and equations, geometry, and statistics and probability. Students received a mastery score for each of these topics based on their current grade level. The test was completely controlled by an outside source and the school could not influence scores or the questions asked (see Appendix A).

Students have been taking this test every six weeks (during the school year) beginning in first grade. They took the test three times during this twelve-week study, once at the beginning, middle (6th week), and end of the study. The scores utilized in this study were the *Scaled Score,* the *Grade Equivalent,* the *Percentile Ranking,* the *Normal Curve Equivalent,* and the *Domain Scores*.

The Scaled Score was “based on the difficulty of the questions and the number of correct responses. The Scaled Score (SS) was useful for comparing student performance over time and across grades” (Renaissance Learning, 2022). These data were used to show students growth compared to themselves, rather than their peers. Students were grouped by their language identifiers: ELLs, SELs, and EOs. Pre-intervention scores from the twelve weeks leading up to the intervention were compared with post intervention scores taken just after the intervention was completed. This measure allowed a comparison between growth not influenced by the intervention and growth heavily influenced by the intervention.

The Grade Equivalency (GE) score was a norm-referenced score that compared the student with other students who took the test recently in the United States (Renaissance Learning, 2022). If a student had a GE of 8.4, that student was scoring similarly to the average eighth grader in the fourth month of the school year. This score did not mean that students should be placed in a different grade level than their current one, simply that they were scoring similarly to the average of that grade level. The number represented the year and month. For example, a GE of 7.5 represented the fifth month in the seventh grade. Students would be expected to grow .3 during the span of 12 weeks. The GE of students for the twelve weeks before the intervention was compared with the GE of students for the twelve weeks during the intervention. Students were again disaggregated by their language identifiers for this measure.

The Percentile Ranking (PR) was the student’s national percentile ranking against their peers who took the test in the same testing window (Renaissance Learning, 2022). A student with a PR of 75 scored higher than 75% of their peers at that time. The PR of students for the start of the intervention was compared with the PR of student just after the intervention. Language groups were again utilized for these data.

The Normal Curve Equivalent (NCE), unlike the PR, worked on an equal-interval scale. Fifty was always the mean, so NCE scores should stay consistent year to year (Renaissance Learning, 2022). Therefore, a positive NCE gain score meant that a student improved more than the average student in their grade level and a negative NCE lose score meant that a student was not growing as quickly as the average student in their grade level. The NCE of students for the twelve weeks before the intervention was compared with the NCE of students for the twelve weeks during the intervention. Once more, students were disaggregated by their language identifiers for this measure.

The Domain Scores (DS) showed the student's mastery of the main CCSS strands for that student's grade level (Renaissance Learning, 2022). The DS for "Expressions and Equations" were utilized most in this study as it was the topic and standards being covered in the regular curriculum at the time of the study. The DS score for all three STAR Renaissance administered during this study were used to compare the average growth over time for each of the three groups.

Summative Student Assessments

Engage New York, sometimes called Eureka Math, was the curriculum utilized at the school site where this study took place. Students in this study had been using this curriculum since kindergarten. Most students had attended the school since preschool. Engage New York was a fully CCSS-aligned curriculum and there was a designated book for each grade level. This was an entirely free and open-source textbook series created by educators in the state of New York to provide a rigorous, CCSS curriculum for every student in the early 2010s. There were six to seven modules per grade level, and although summative tests were provided by the curriculum, they were not utilized for summative assessments in my math class. Summative tests were cultivated from the exit tickets provided by the curriculum or self-created, so that each question or set of questions specifically addressed one standard at a time (see Appendix B). Summative assessments were given two to three times a module and usually included one to three standards. Students were notified of the test day a few weeks in advance and were given the topic and general set-up of every question on the test two days before the test, as well as the corresponding standard for each question on the test. In this research study, students took approximately 5 summative assessments, which mainly covered algebraic expressions and equations. Three of the five assessments were utilized in this study, due to the nature of the questions asked on each assessment. These data were the scores from one problem from each assessment, which asked students to explain their thinking.

Summative assessments were scored on a modified Marzano Scale. One score was given for each problem or the set of problems that correspond to one standard. If the student showed full mastery of the problem or set of problems, and therefore the standard, they earned 3 points for that problem or set of problems. If the student showed strong understanding but could not explain their work or made a small error, they earned 2.5 points for that problem or set of problems. If the student showed some understanding or inconsistent understanding, they earned 2 points for that problem or set of problems. If the student showed some little or no understanding, they earned 1.5 or 1 point for that problem or set of problems. These points were then averaged, and students were given a mastery score out of 3. This score was then translated to a score out of 10 to better fit traditional grading scales. A 3 translated to a 10 out of 10, a 2.5 translated to an 8.5 out of 10, a 2 translated to a 7.5 out of 10, a 1.5 translated to a 6.5 out of 10, and a 1 translated to a 5.5 out of 10. A zero meant that a student did not attempt to answer the question in anyway. However, I did not allow students to leave problems blank, so every student at least attempted each problem. Incremental scores were included in both scoring scales.

Summative assessments were comprised of a majority of open response questions. Some of these questions ask for written explanations. I have observed that many of my students struggle to explain their reasoning in a written response, although they may be able to perfectly complete a purely numerical question of the same topic or concept. There are no written response questions on the STAR Renaissance Standardized test, therefore, summative assessments uniquely provide the qualitative data of written explanations in a formal setting, as opposed to in class work, which was also collected for this study.

Audio Recordings of Student Mathematical Discussions

Because mathematical discussions were a central focus of this study, audio recordings were taken during these discussions. Large group conversations as well as group discussions were recorded once a week and transcribed with all identifiers replaced with pseudonyms. These data were used qualitatively to assess student understanding and use of the mathematical register. Audio recordings were utilized in conjunction with student work to compare students' oral and written responses during a given lesson. Students and families were notified that audio recordings would be taken during mathematical discussions, but due to the small size of the recorder, the students were not always aware that the device had been activated.

Student Work

Student work samples were archived for analysis to assess student understanding and engagement during discussions. Student work samples were taken after students had completed a mathematical discussion centered around that problem or set of problems. Student work was recorded for qualitative analysis in conjunction with the audio recordings.

The Engage New York middle school curriculum rarely provided direct examples in the student book. The lessons usually started with a few problems that the teacher guides students through and was then followed by practice problems of a similar nature for students to try on their own. Typically, questions were posed and then a large open space was provided for students to take notes and work out the problem. There were sometimes models that accompany a problem, but there were never pictures or showy asides to engage students. Student copies were printed in black and white. Problems frequently included a diverse array of names, were age-appropriate, and were culturally and social-economically aware. Engage New York problems were often complex and did not have neat or simple answers. Critical thinking was frequently expected of students. The curriculum fully embodied the Common Core drive for rigorous and deeper learning (see Appendix C).

Procedures

The study took place over twelve school weeks beginning in December 2021 and concluding at the end of February 2022. There was a three week break in the study for winter vacation. Mathematical discussions were very time consuming, so they were utilized once a week during the course of this study. Mathematical discussions were used when most appropriate. Discussions focused around one problem or a set of related problems. Problems or a set of related problems were chosen in part based on their complexity. A simple and straightforward problem would not have provided space for students to have detailed discussions. Problems or a set of related problems were also chosen as the focus for discussions based on lessons that were most closely focused on mastering a standard, rather than building up to or extending the standard. I was mindful in my procedure that although middle school students often crave autonomy, in my experience, they strongly dislike being asked to complete exploratory work in which they are given little or no direction at the start of the lessons. I aimed to strike a balance in my approach.

The following details the full procedure followed when the mathematical discussions were implemented. This procedure occurred roughly once a week in place of a teacher-led and direct instruction focused lesson.

Introduction

The lessons began with the students' being introduced to or reminded of any pertinent words, phrases, or understandings from the mathematical register, both verbally and visually. If something new was being introduced, I worked to tie it to something already know by the students. For example, when introducing the idea of balancing an equation, I taught the student "what you do to one side, you must also do to the other". I equated this to finding equivalent fractions and remind students of the previously learned phrase: "what you do to the top, you must also do to the bottom". I associated the procedure of applying an operation twice to the same problem, but in different locations to balance the numbers. By doing this, I am utilizing Lev Vygotsky theory of scaffolding, or building on previous knowledge (Vygotsky, 1986).

Example 1

Next, I moved through an example problem or two via a document camera using the student book. I frequently paused to ask if students had any questions. If they did, I asked them to be specific and, if need be, to identify the last step that made sense to them before they became confused. (This is a norm I established at the start of the school year and students were aware of the expectation). I expected that students wrote down what I was writing and to add any notes that they thought would be helpful to them.

Mathematical Discussions

Then, students were instructed to complete the following problem or set of problems with their preassigned group. Groups were built into the seating chart. Groups were made up of three students, one identified as ELL, one as SEL, and one as EO. Some groups had two ELLs and one EO as there were slightly fewer SELs overall. Each member of the group self-assigned a job before beginning their discussion. Students could choose to be the leader, the recorder, or the gatekeeper. The leader made sure every voice in the group was heard, the recorder took detailed and neat notes, and the gatekeeper kept everyone on task and redirected any off-topic behavior. All three students regardless of their job were expected to participate in the discussion equally and to take comprehensive notes. Expectations were discussed at the start of the study and students were reminded of the duties of each role when moving into discussion groups (see Appendix D). The teacher circulated the room during discussions, answering questions and redirecting student behavior. The audio of these conversations was recorded for later analysis.

Class Share Out

Next, I asked the recorder to share their notes via the document camera with the large group. I asked one of the group members to explain to the class the process they used to work through the problem based on their group discussion. I prompted them to use the mathematical register in their explanations if they did not. I did not confirm the correct answer to the problem(s) until each group had shared out their process. I emphasized that there are many ways to solve a problem, so the book’s method may have differed from their own and that that was perfectly acceptable as long as they showed their work. I made copies of the student work that students shared out for later analysis.

These share outs often led to further large group discussions as students asked often asked clarifying questions during student share outs. This was an organic development in the procedure but proved helpful to student understanding.

Consensus and Large Group Discussions

Finally, as was originally intended, large group discussions followed after the share outs when the class had come to a consensus on the answer(s) to the problem(s) and I clarified misunderstandings present in the share outs or small group discussions. Students raised their hands to share insights about the problem(s) or ask clarifying questions during this stage of the intervention. The audio of these conversations was also recorded for later analysis.

Plan for Data Analysis

Data was collected to answer the question: *How do mathematical discussions impact middle school students' deeper understanding of mathematical concepts, particularly for ELLs?* I triangulated the data by using multiple sources of data, including summative tests, standardized STAR tests, audio recordings of mathematical discussions, and student class work. This multitude of data allowed for several approaches at answering the above question to ensure data was accurate and bias was minimized.

Quantitative data was collected from summative tests and standardized STAR tests. Summative assessments are scored on a modified Marzano Scale (3 to 1) and the main STAR scores utilized in this study were the *Scaled Score,* the *Grade Equivalent*, the *Percentile Ranking*, the *Normal Curve Equivalent*, and the *Domain Scores*. Data were disaggregated via language subgroups: ELLs, SELs, and EOs. These scores were compared using a group average from data at different points before, during, and after the study. Data for some measures were also presented as whole class averages comparing students prior to and after the study.

I collected qualitative data through audio recordings and student class work. The recordings were transcribed, and names were replaced with pseudonym to maintain anonymity. The use of mathematical language and the mathematical register were of particular interest in this study and quotations from audio recordings were used to show the development of mathematical language acquisition over the course of the study. This data was particularly impacted by the COVD-19 requirements present during this study and by the limiting sound quality of the audio recordings. Collecting data from the audio recordings was proved especially challenging.

Summary

The goal of this action research project was to explore the use of mathematical discussions to impact middle school students' deeper understanding of mathematical concepts, particularly for ELLs. I observed that my middle school student struggled to communicate deeper understanding effectively, and informed by extensive research, attempted to impact this academic challenge through mathematical discussions. This study focused on mathematical discussions, which were structured so groups of students varied in their language levels (ELL, SEL, EO). After seeing an example or two led by the teacher, students self-assigned roles for their mathematical discussion to keep the discussions productive (leader, recorder, gatekeeper) (see Appendix D). If time allowed, students also shared out their small group ideas to the larger group. Summative student assessments, STAR Renaissance standardized tests, audio recordings of students' mathematical discussions, and students' class work were used to measure student understanding and growth over the course of the study.

This chapter discussed the setting, the participants, the data collection strategies, the procedures, and the plan for data analysis. The following chapter will focus on the analysis of the data that was collected in this study.

Chapter IV

Findings

The purpose of this action research project was to help deepen my middle school students' mathematical understanding through mathematical discussions, with a focus on my English Language Learners (ELLs). Therefore, the action research question that guided this study was: *How do mathematical discussions impact middle school students' deeper understanding of mathematical concepts, particularly for English Language Learners?* In my own classroom, I have observed that students struggle to explain their thinking in both written and oral formats. They understand the mathematical concepts enough to solve number-only problems accurately, but struggle in their attempts to explain their work using mathematical language, or what some researchers term, the mathematical register. I believed mathematical discussion would be highly engaging and beneficial for my students, especially my ELLs.

The reviewed research documented that mathematical discussions were a recognized mode for students of all language levels to actively participate in robust mathematical learning, as well as, providing students ownership in their own learning (Moschkovich, 2012; Sigley and Wilkinson, 2015; Zahner et al., 2012). This research was situated in the larger findings of Lev Vygotsky's zone of proximal development (Vygotsky, 1978) and Stephen Krashen and Tracy Terrell's natural approach to language acquisition (Krashen and Terrell, 1998), who both theorized that learning a new skill occurs through the natural interaction between a learner and a more knowledgeable person.

Utilizing the mathematical register can be challenging for all students due to its frequent use of new vocabulary and the use of familiar words that have different mathematical meanings (Schleppegrell, 2007). However, using the mathematical register can be particularly difficult for

ELLs and Standard English Learners (SELs) (AEMP, 2021). Because reading scores explain 44–54% of the variance in Math scores (Chen and Chalhoub-Deville, 2016, p.583), mathematics can be a gatekeeping subject for students, especially ELLs and SELs (Howard, 2020, p.16,18). Dr. Judit Moschkovich is a strong proponent of the practice of mathematical discussion, as this practice holds space for students to be sources of knowledge, rather than passive receivers of curated information (Moschkovich, 2007). This is not only engaging for young people, but also empowering for students who are often marginalized by our modern school system. I hope to expand this research to my middle school students and extend this methodology in relation to SELs.

This chapter begins with an overview of the methods used in this action research project and summarizes how the data was collected. Then, the demographics of the participants are detailed, followed by thorough analyses of each of the data collection methods: STAR Renaissance Standardized Tests, Audio Recordings of Mathematical Discussions and Student Classwork, and Summative Student Assessments. This chapter concludes with a summary of the findings of this study.

Overview of Methods and Data Collection

To complete this action research project, data were collected over a twelve-week period. Mathematical discussions occurred once a week in conjunction with regular classroom curriculum. Discussions focused around one problem or a set of related problems which were complex enough to encourage comprehensive conversation. I also chose problems that were most closely related to mastering of a standard, rather than building up to or extending a standard. Students were grouped in sets of three: one ELL, one SEL, and one English only learner (EO). Each student in the group took on a role: leader, recorder, or gatekeeper. Students rotated through the different roles over the course of the study (see Appendix D). The leader was instructed to ensure that every voice in the group was heard. The recorder was instructed to take the neatest notes, although everyone in the group was asked to take appropriate notes. The recorders notes were shared out with the large group after the small group discussion. The gatekeeper was instructed to keep everyone on task and to redirect students who may have become distracted or off-topic. Students were particularly engaged by this role. The students then shared out their work. Lastly, a consensus about the answer to the problem was discussed in the large group. During this time, students also asked clarifying questions and I explained any misunderstandings present in the share outs or small group discussions.

A variety of qualitative and quantitative data upholds the validity of the results obtained in this study and were gathered from the following sources: STAR Renaissance standardized tests, audio recordings of students' mathematical discussions, summative student assessments, and students' class work. All identifiers were removed and replaced with pseudonyms for this study.

Demographics of Participants

The participants of this study were from my seventh-grade class during the 2021-2022 school year. The small group of pullout students in this grade level did not participate in this study. There were 26 students total in this study who were considered general education or high-achieving learners. About 46% of these students identified as young women and 54% identified as young men. Approximately, 73% of the participants identified as Asian or Pacific Islander, which included Chinese, Japanese, and Vietnamese, but predominately Filipino. Approximately, 20% identified as Two or More Races, and less than 10% identified as Hispanic or Latino, less than 10% identified as Black or African American, and less than 10% identified as White or

Caucasian. No students in this group received special educations services and less than 10% qualified for free or reduced lunches.

Though not formally identified at my private school, I have observed that about a third of students spoke another language at home some, most, or all of the time. Another third of the students regularly communicated with parents or grandparents whose first language was a language other than English. The final third of students spoke only English at home and had parents or grandparents whose first language was also English. These three groups were identified in this study as ELLs, SELs, and EOs, respectively. Daily instruction and student discussions were conducted fully in English.

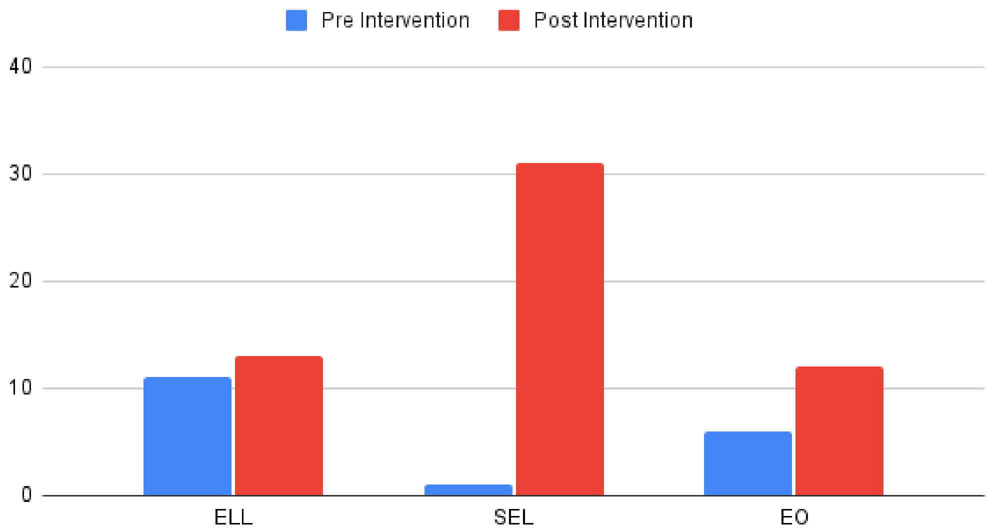
Analysis of STAR Renaissance Standardized Tests

STAR Renaissance is a national standardized test that evaluated students' mastery of Common Core State Standards (CCSS) and compared them both to an internal scaled score and to other students who took the test recently in the United States. The test was completely controlled by an outside source and the school could not influence scores or the questions asked (see Appendix A). Student participants took this test three times over the course of this study, once at the beginning, middle (6th week), and end of the study. The scores utilized in this study were the *Scaled Score*, the *Grade Equivalent*, the *Percentile Ranking*, the *Normal Curve Equivalent*, and the *Domain Scores*.

The Scaled Score was "based on the difficulty of the questions and the number of correct responses. The Scaled Score (SS) was useful for comparing student performance over time and across grades" (Renaissance Learning, 2022). These data were used to show students growth compared to themselves, rather than their peers. Figure 1 shows the group averages of the scaled scores for students identified in this study as ELLs, SELs, and EOs. Pre-intervention scores showed that during the twelve weeks leading up to the intervention, students averaged one to eleven points of growth within their respective groups. After the intervention, students showed averages of 12 to 31 points of growth. All groups made more growth on average during the intervention, however, SELs made significantly more than the other groups. ELLs and OEs grew 2 and 6 additional points, respectively, but SELs grew 30 additional points.

Figure 1

Average Scaled Score Growth

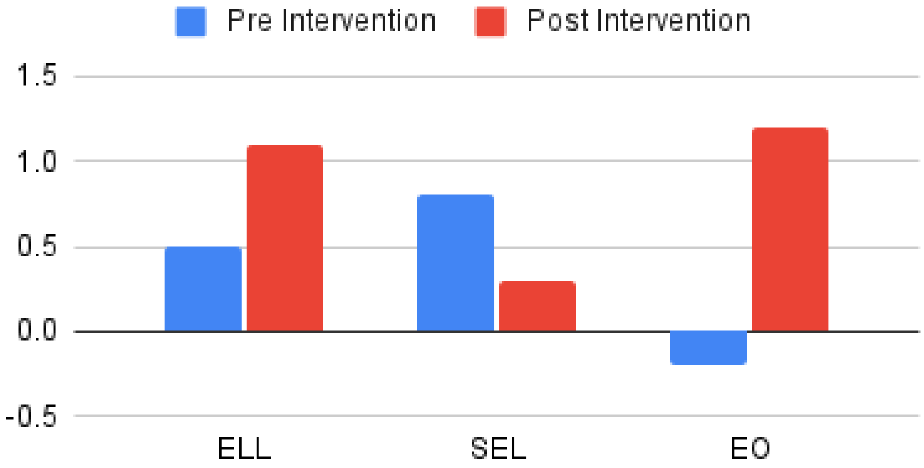


Note. Average scaled scores for students identified as ELLs (*n*=10), SELs (*n*=7), and EOs (*n*=9) before and after the intervention (*N*=26).

The Grade Equivalency (GE) score was a norm-referenced score that compared the student with other students who took the test recently in the United States (Renaissance Learning, 2022). If a student had a GE of 8.4, that student was scoring similarly to the average eighth grader in the fourth month of the school year. This score did not mean that students should be placed in a different grade level than their current one, simply that they were scoring similarly to the average of that grade level. The number represented the year and month. For example, a GE of 7.5 represented the fifth month in the seventh grade. Figure 2 shows the average growth for students in the three previously mentioned categories. Students would be expected to grow .3 during both the pre- and post-intervention as both periods of time spanned 12 weeks. However, during the intervention, ELLs grew 1.1 and EOs grew 1.2, far exceeding the expected growth and far more than their own growth during the pre-intervention window. However, SELs showed more growth during the pre-intervention (.8) and average growth during the intervention (.3).

Figure 2

Average Grade Equivalent Growth

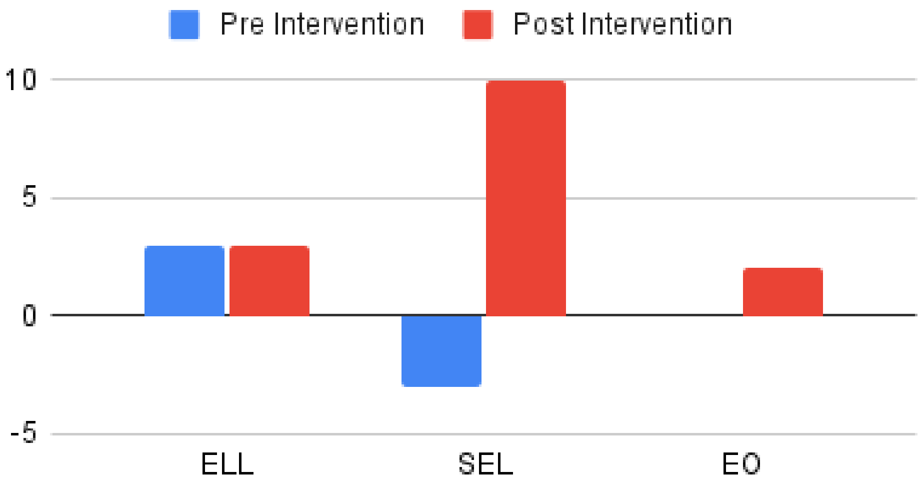


Note. Average grade equivalents for students identified as ELLs (*n*=10), SELs (*n*=7), and EOs (*n*=9) before and after the intervention (*N*=26).

The Percentile Ranking (PR) was the student's national percentile ranking against their peers who took the test in the same testing window (Renaissance Learning, 2022). A student with a PR of 75 scored higher than 75% of their peers at that time. Figure 3 shows the average PR for students in their respective groups. ELL students showed the same ranking before and after the intervention (3%). SEL students showed a marked increase in their PRs (-3% to 10%) and EOs showed a small increase in their scores (0% to 2%).

Figure 3

Average Percentile Ranking Growth

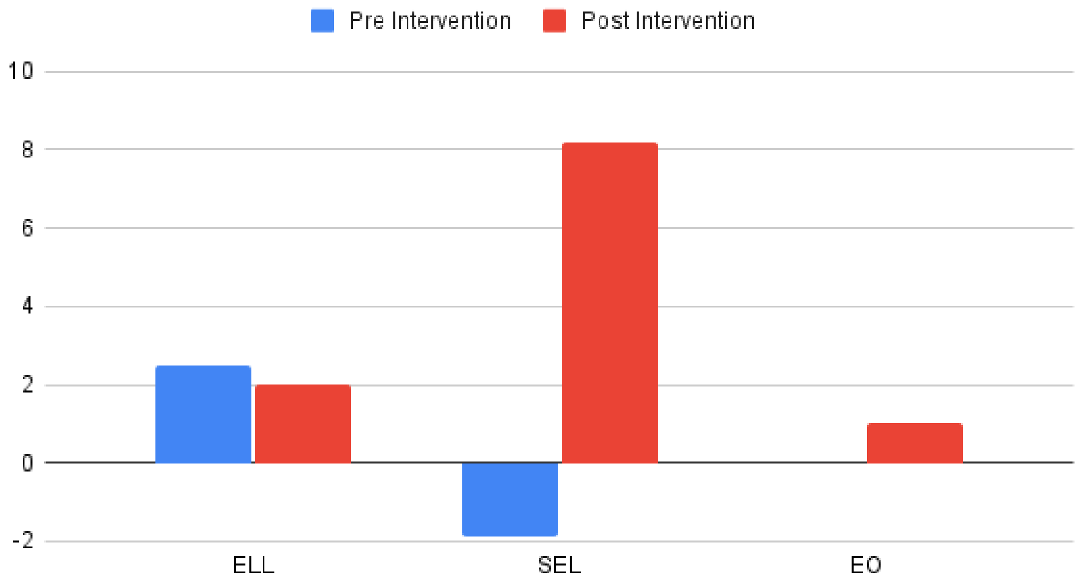


Note. Average percentile ranking for students identified as ELLs (*n*=10), SELs (*n*=7), and EOs (*n*=9) before and after the intervention (*N*=26).

The Normal Curve Equivalent (NCE), unlike the PR, worked on an equal-interval scale. Fifty was always the mean, so NCE scores should stay consistent year to year (Renaissance Learning, 2022). Therefore, a positive NCE gain score meant that a student improved more than the average student. All groups showed positive NCE scores, meaning that all groups grew more than was expected. During the intervention, ELLs grew 2 points (compared to 2.5), SELs grew 8.2 point (compared to -1.9), and EOs grew 1 point (compared to 0). SELs grew far more than the other groups, gaining almost 10 points compared to their peers.

Figure 4

Average normal Curve Equivalency Score



Note. Average normal curve equivalent for students identified as ELLs (*n*=10), SELs (*n*=7), and EOs (*n*=9) before and after the intervention (*N*=26).

A summation of the above data is shown in Table 1 without grouping categorization. Overall, every category showed substantial growth. Students overall averaged 10 more points on their Scaled Score, 4 months more of Grade Equivalent score, 4% higher Percentile Scores, and 2.5% more growth on their Normal Curve Equivalent scores.

Table 1

Pre- and Post-Intervention Averages for Whole Group

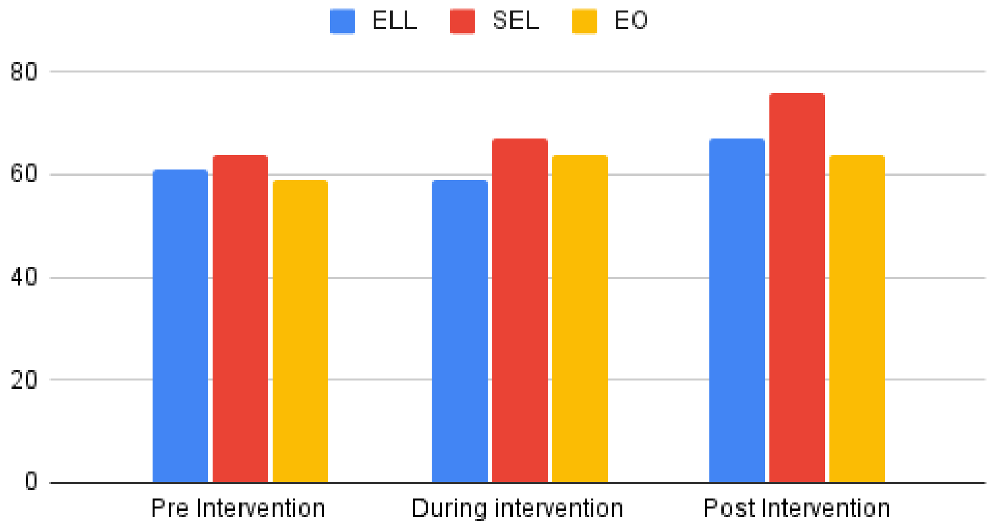
	Average Scaled Score Growth	***Average Grade Equivalent Growth***	***Average Percentile Ranking Growth***	***Average Normal Curve Equivalent Growth***
Pre-Intervention (12 weeks of growth prior to intervention)	8	5 months	1	.9
Post Intervention (12 weeks of growth during the intervention)	18	9 months	5	3.4

Note. Averages for all students regardless of identification before and after the intervention (*N*=26).

The Domain Scores (DS) showed the student's mastery of the main CCSS strands for that student's grade level (Renaissance Learning, 2022). The DS for "Expressions and Equations" were utilized most in this study as it was the topic and standards being covered in the regular curriculum at the time of the study. Figure 5 shows the average score for the three groups before, during, and after the study. These data were collected in 6-week intervals. All groups show eventual growth from start to finish. However, the EO group showed more growth in the first six weeks (5%) than the second 6 weeks (0%) and the ELLs showed the reverse (-3% and 8%, respectively). The SEL groups grew at a more consistent positive rate of the three groups, growing on average 3% in the first six weeks and 9% in the second six weeks. SELs showed the most substantial average growth overall, although all groups showed positive growth from the beginning to the end of the intervention of mathematical discussions.

Figure 5

Average Expressions and Equations Domain Scores



Note. Average domain scores for expressions and equations for students identified as ELLs (*n*=10), SELs (*n*=7), and EOs (*n*=9) before and after the intervention (*N*=26).

Analysis of Audio Recordings of Mathematical Discussions and Student Classwork.

Because mathematical discussions were a central focus of this study, audio recordings were taken during these discussions. Large group conversations as well as small group discussions were recorded once a week and transcribed with all identifiers replaced with pseudonyms. Table 2 shows quotes of students during the first and the tenth discussions. During the first mathematical discussion, none of the students were able to provide a robust explanation. However, by the tenth mathematical discussion, all students were able to give longer and more complex justifications. Students were also able to use newly introduced vocabulary more frequently in their later mathematical discussions and many students cited evidence from the problem or the lesson in their explanations, rooting their justification in the problem being solved, rather than relying on outside, personal information.

In the first recording, Student A used reasonability to explain their answer, but quickly stated that they were not sure why they felt that way even though they knew they should continue their explanation. By the tenth recording, Student A is citing direct examples from the problem to justify their answer. They even end their explanation with the definition of the current topic (functions) in conjunction with the current problem.

Student B at first offered justification for their answer by citing examples outside of the presented problem. They also fell back on Student A's explanation of reasonability, but again did not provide evidence to support this assertion. Later, Student B cited the learning expectation of the lesson (denoted in the lesson summary) and tied it to the current problem. Their explanation was not as specific as Student A's, but it was markedly more concrete than their explanation during the first recording session.

Initially, Student C is confused by the initial question. They clarified that there were no numbers involved in solving the problem, only words. Student C actually answered correctly, but only partially explained their thinking, saying that "…it's going to be a lot of grams. So, ya…". By the tenth recording session, Student C showed more confidence and stamina. They told another student in their group, "If you can't solve it, how do you know what it is, and then you won't be able to explain it. You got to figure it out." They again were one of the first students to correctly answer the problem, however, this time they used precise vocabulary words to explain themselves, i.e., "input" and "output".

Student D originally showed a connection to the problem in their answer, but like Student C, did not provide a robust explanation. Near the end of the study, Student D was able to cite evidence from other mathematical concepts to justify their answer. They connected the reversibility of earlier learned concepts and applied that conceptual understanding to this new topic. They were also able to utilize the guiding definition and vocabulary of the current lesson in their explanation, similarly to the other students.

Though only a small sample, these quotes illustrate the growth that students made over the course of the study. Students notably provided longer and more thoughtful answers. They all used vocabulary, definitions, or broader mathematical concepts far more frequently to justify their answers.

Table 2

Sample Explanation Quotes From 1st Recording and 10th Recording

	Student A ***EO***	***Student B*** ***SEL***	***Student C*** ***ELL***	***Student D*** ***EO***
First Recoding	*"Because it sounds more reasonable… because…I didn't get that far."*	*"One because Spotify does it. Two because seconds...um...that just doesn't make sense. And years… I don't think it can hold that many years. So, days."*	*"So, this is it? For the whole exercise, we just have to write the why?... So, there's no numbers?"* *"I pick pounds, because it's going to be a lot of grams. So ya…"*	*"I was going to say grams because it's already in grams, so why not."*
Tenth Recording	*"It's the same thing as the first one, because there's two inputs of 7, but their output is 15 and 10…the inputs aren't unique and they should be the same, which makes the function wrong. The 7 inputs do not have the same unique output."*	*"It says in the lesson summary that each input has to have one unique output. But these are different inputs, so I don't think it would really matter what the output would be. That's just what I read…"*	*"There are two fives here, so it is not [a function] … So, each one is supposed to be unique. So, each input and output should not be the same."* *"The outputs are the same on this one, but the inputs are the different. Bro, does that really matter?"* *"If you can't solve it, how do you know what it is, and then you won't be able to explain it. You got to figure it out."*	*"We were thinking that it's no, because the output is the same and input is different, so it should still be no. Because it's like multiplication and division, they are opposites of each other. Like addition and subtraction are opposites of each other. So, the input being the same and the output being different, then it should still be the same thing."*

Note. Samples of explanatory quotes from first recording and tenth recording.

Table 3 shows student work samples which were also archived for analysis to assess student understanding and engagement during discussions (see Appendix C). Student work samples were taken directly *after* students had completed a mathematical discussion. These samples show the similar progression of students' ability to use vocabulary and definitions in their explanations over the course of the study even when the answer was known to them.

Table 3

Classwork From 1st Recording Session and 10th Recording Session

	Student E ***SEL***	***Student F*** ***ELL***	***Student G*** ***EO***
Classwork from First Recoding Session	*"It would be pounds because if you're starting with grams you need to convert it to pounds."*	*"I think pounds because grams would be too small and there isn't enough sugar for tons."*	*"Pounds because grams would be too big of a number and tons would be too big of a fraction."*
Classwork from Tenth Recording Session	*"No because each input should have a unique output and there are two inputs with the same output."*	*"There is no function because 2 different inputs have the same output."*	*"There is no function because one input has different outputs (not unique)."*

Note. Samples of explanatory classwork from first recording session and tenth recording session.

Analysis of Summative Student Assessments

Engage New York, sometimes called Eureka Math, was the fully CCSS-aligned curriculum utilized at the school site where this study took place. Summative assessments were given five times during this study and each included two to three standards. Three summative assessments were used to collect data, one for the beginning, middle, and end of the study. (see Appendix B).

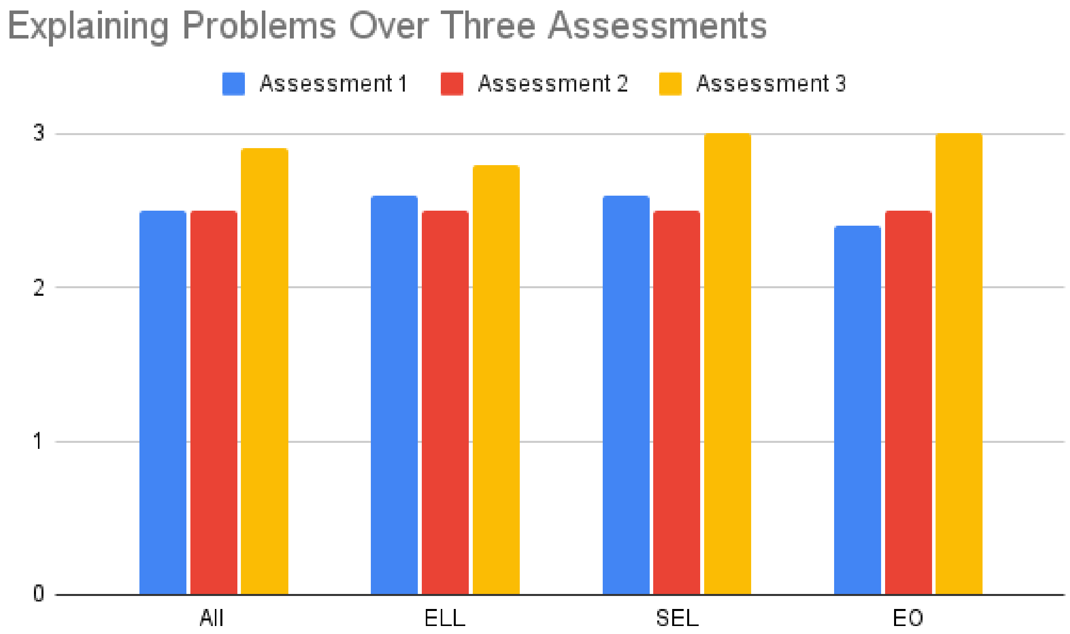
Summative tests were scored on a modified Marzano Scale. One score was given for each problem or the set of problems that correspond to one standard. If the student showed full mastery of the problem or set of problems, and therefore the standard, they earned 3 points for that problem or set of problems. If the student showed strong understanding but could not explain their work or made a small error, they earned 2.5 points for that problem or set of problems. If the student showed some understanding or inconsistent understanding, they earned 2 points for that problem or set of problems. If the student showed little or no understanding, they earned 1.5 or 1 point for that problem or set of problems. A zero would mean that a student did not attempt to answer the question in anyway. Incremental scores were included in this scoring.

I have observed that many of my students struggle to explain their reasoning in a written response, although they may be able to perfectly complete a purely numerical question of the same topic or concept. There were no written response questions on the STAR Renaissance Standardized test, therefore, summative assessments uniquely provided the quantitative data of written responses in a formal setting.

Figure 6 compares the averages of the three student groups identified in this study, as well as the class as a whole. Three of the five assessments were utilized in this study, due to the nature of the questions asked on each assessment. These data were the scores from one problem from each assessment, which asked students to explain their thinking. As seen in Figure 9, most students were able to answer the assessment question accurately, but only earned 2.5 points, because they did not explain their thinking appropriately. This held true for both the first and second assessment, although the EOs made slightly more growth than the other groups on the second assessment. However, all students showed improvement by the third assessment, despite the ELLs making slightly less growth than the other groups. When further analyzing the responses on the third assessment, most students were able to answer correctly and explain their thinking adequately. However, only two students (one ELL and one SEL) used the newly introduced vocabulary word in their explanation.

Figure 6

Explaining Problems Over Three Assessments



Note. Average scores on explanatory problems over three assessments for students identified as ELLs (*n*=10), SELs (*n*=7), and EOs (*n*=9) before, during, and after the intervention (*N*=26).

Summary

The purpose of this action research project was to help deepen my middle school students' mathematical understanding through mathematical discussions, with a focus on my ELLs and SELs. This study took place over a twelve week period of time. Data were gathered from the following sources to determine the effectiveness of the intervention: STAR Renaissance standardized tests, audio recordings of students' mathematical discussions, summative student assessments, and students' class work.

Quantitative data were collected via the three STAR Renaissance standardized tests and the three summative assessments that students took over the course of this project. Qualitative data was collected via audio recordings of students' mathematical discussions, and students' class work. When integrating the analysis of these sources of data, I determined that mathematical discussions deepen middle school students' mathematical understanding, especially for ELLs and SELs. However, this intervention was more success for SELs, than their ELL counterparts.

In the next chapter, I discuss these findings of this study and investigate the implications of this action research project. I will also resituate my findings in the context of the existing research previously discussed in the literature review. The next chapter will also expand on my plans for future interventions in my own classroom and provide potential avenues that other scholars and educators might explore in the future based on my findings.

Chapter V

Conclusions

The Common Core State Standards (CCSS), in an effort to better prepare students to be college and career ready, focus heavily on deeper understanding pursuing “conceptual understanding, procedural skills and fluency, and application with equal intensity” (National Governors Association Center for Best Practices, 2010). These standards emphasize the importance that communicating mathematically is much greater than communicating *what* the answer is, but equally important is *how* one arrives at the answer. This deeper level of thinking can be challenging for all students but may be especially challenging for students who are English Language Learners (ELLs) or Standard English Learners (SELs). There is a strong relationship between English language proficiency and math achievement (Chen, F., and Chalhoub-Deville, M., 2016), so educators must be especially mindful of the impact of this shift of focus for ELLs and SELs.

In order to meet the rigor of the CCSS and its demanding and relentless focus on the student’s ability to answer the question adequately and articulately: why? Dr. Moschkovich writes that English Language Learners can and should participate in mathematical discussions regardless of their English proficiency (Moschkovich, 2012, p.305). English Language Learners do not need to be spoon-fed mathematical language and that this practice can actually negatively impact the achievement scores compared to students who were given more agency in their math classes (Zahner et al., 2012). If we see students as sources of knowledge and valued contributors to class discussion, we will also see mathematical achievement scores rise and historical achievement and opportunity gaps reduced.

In my own classroom, I have witnessed the struggle students have when answering Common Core questions that require justification and reasoning, especially among my ELLs, who account for approximately a third of my students, and my SELs, who account for another third of my classroom. I believed that mathematical discussion would be highly engaging and beneficial for my students and allow them to bring to the lesson new insights that would be lost if only direct instruction were implemented. I believed these discussions would also afford students the agency and independence that most middle school students are beginning to crave. I hoped that this technique would elevate and engage my students, especially my ELLs and SELs. Therefore, the purpose of this action research project was to help deepen my middle school students' mathematical understanding through mathematical discussions, with a focus on my ELLs and SELs, and the action research question that guided this study was: *How do mathematical discussions impact middle school students' deeper understanding of mathematical concepts, particularly for English Language Learners?*

In Chapter IV, the data collected during this study were examined and trangulated. These data show that mathematical discussions do have a positive impact on students deeper understanding, although the effects were most positive for SELs by most of the measurements taken. The following is a summary of these findings focused in three areas of measurement, which is followed by an interpretation of the findings, limitations of this study, a comprehensive summary of the action research project as a whole, and a plan for future action based on the conductings this study.

Summary of Findings

To complete this action research project, data were collected over a twelve-week period to determine if mathematical discussions had a positive effect on student's deeper understanding, especially for my ELLs and SELs. A variety of qualitative and quantitative data upholds the validity of the results obtained in this study and were gathered from the following sources: STAR Renaissance standardized tests, audio recordings of students' mathematical discussions, summative student assessments, and students' class work. Twenty-six students, who were all considered general education or high-achieving learners participated in mathematical discussions, which occurred once a week in conjunction with regular classroom curriculum. Students were grouped in sets of three, one ELL, one SEL, and one English only learner (EO), and each student in turn took on a role: leader, recorder, or gatekeeper (see Appendix D). After twelve weeks, the data collected was de-identified and analyzed.

Analysis of STAR Renaissance Standardized Tests

The STAR Renaissance provides numerous measurements to evaluates students. All measures showed student growth over the course of the study, although some were more conclusive than others. Compared to the 12 weeks leading up to the study, students averaged 10 more points on their Scaled Score, 4 months more of Grade Equivalent score, 4% higher Percentile Scores, and 2.5% more growth on their Normal Curve Equivalent scores. However, disaggregating the data showed that SELs made the most growth of the three subgroups (Figure 1-5).

The Scaled Score, which was used to show students growth compared to themselves, indicated that during the twelve weeks prior to the intervention, students averaged 1 to 11 points of growth within their respective groups. After the intervention, students showed averages of 12 to 31 points of growth. ELLs increased by 2, SELs increased by 30, and EOs increased by 6. While all groups made more growth on average during the intervention, SELs made significantly more growth than the other groups.

The Grade Equivalency scores, which compared the student with other students who took the test recently in the United States, showed that all student groups averaged expected or more than expected growth for the twelve weeks prior to and during the study (Figure 2). During the intervention, ELLs grew 1.1 (compared to .5), SELs grew .3 (compared to .8), and EOs grew 1.2 (compared to -0.2). This data set did not match the other sub scores, which showed SELs making the most growth.

The Percentile Ranking, the student's national percentile ranking against their peers who took the test in the same testing window, showed that on average students in their respective groups, matched or exceeded their Percentile Ranking growth from the twelve weeks leading up to the intervention. Both before and during the intervention, ELLs on average grew 3%. SELs on average grew 10%, compared to a negative growth of 3% prior to the intervention. EOs grew on average 2% during the study after not showing any growth in this sub score prior to the study. Again, SELs showed the most substantial growth by this measure.

The Normal Curve Equivalent scores should stay consistent year to year. Therefore, a positive NCE gain score meant a student was improving more than the average student (Renaissance Learning, 2022). All groups showed positive NCE scores, meaning that all groups grew more than was expected (Figure 4). During the intervention, ELLs grew 2 points (compared to 2.5), SELs grew 8.2 point (compared to -1.9), and EOs grew 1 point (compared to 0). SELs showed the most considerable growth in this category.

The Domain Scores showed the student's mastery of CCSS "Expressions and Equations" standards. These data demonstrated that all groups on average show eventual growth from the start of the study to its conclusion. However, the EO group showed more growth in the first six weeks (+5%) than the second 6 weeks (0%) and the ELLs showed the reverse (-3% and 8%, respectively). The SEL group grew at the most consistent rate of the three groups, growing on average 3% in the first six weeks and 9% in the second six weeks. Once more, SELs showed the most extensive average growth overall, although all groups showed positive growth from the beginning to the end of the intervention of mathematical discussions.

Analysis of Audio Recordings of Mathematical Discussions and Student Classwork

Because mathematical discussions were a central focus of this study, audio recordings were taken during these discussions, were transcribed, and were then de-identified. Student work samples were also archived for analysis to assess student understanding and engagement during discussions. Samples of students spoken and written responses during the first and the tenth discussions showed quantitative growth over the course of the study (Table 2 and 3). Though only a small sample, these quotes and excerpts illustrated the growth that students made over the course of the study. During the first mathematical discussion, none of the students provided a robust explanation in either their discussions or their classwork. However, by the tenth mathematical discussion, all students were able to provide significantly longer and more complex answers. They notably used vocabulary, definitions, or broader mathematical concepts to justify their answers in their later discussions and classwork. In the later discussion, many students cited evidence from the problem, the lesson’s learning expectation, or previously learned concepts in their explanations, rather than relying on outside, personal information or simply not providing any justification at all as they did in the first discussion. Students were far better able to provide robust explanations, showing deeper understanding, from the first to the tenth mathematical discussion.

Analysis of Summative Student Assessments

Summative assessments were given five times during this study and each included two to three standards. Three summative assessments were used to collect data, one for the beginning, middle, and end of the study and were scored on a modified Marzano Scale. I have observed that many of my students struggle to explain their reasoning in a written response, although they may be able to perfectly complete a purely numerical question of the same topic or concept. There are no written response questions on the STAR Renaissance Standardized test, therefore, summative assessments uniquely provided the quantitative data of written responses in a formal setting. The data collated for this measure used one question per assessment, which asked students to explain their thinking. When compared, the averages of the three student groups as well as the class as a whole showed improvement over the course of this study (Figure 6). On this first assessment, most students were able to answer an assessment question accurately, but only earned 2.5 points, because they could not explain their thinking. This held true for both the first and second assessment, although the EOs made slightly more growth than the other groups on the second assessment (0.1). However, all students showed improvement by the third assessment, despite the ELLs making slightly less growth than the other groups (2.8 compared to 3). When further analyzed, the written responses on the third assessment showed that most students were able to answer the question correctly and explain their thinking adequately. However, upon deeper analysis, only two students (one ELL and one SEL) used the newly introduced vocabulary word in their explanation.

The quantitative data collected via the three STAR Renaissance standardized tests and the three summative assessments, and the qualitative data collected via audio recordings of students' mathematical discussions and students' class work showed the effectiveness of using mathematical discussion to deepen middle school students understanding of mathematical concepts. However, these data showed that although the intervention was successful overall, it was more successful for SELs than their ELL or EO conterparts.

These qualitative and quantitative findings connected to and extended the understandings presented in the literature review. The work of Chen and Chalhoub-Deville (2016), Schleppergell (2007), Moschovich (2007), Song and Coppersmith (2020), and Martin (2015) supported the use of mathematical discussions as a ameaningful intervention to more equitably serve ELLs and SELs in the mathematics class. The data of this study additionally maintained the validity and effectiveness of this educational practice in a linguistically diverse classroom.

Interpretation of Findings

The integrated analysis of this study, supported by the existing literature, determined that mathematical discussions deepened middle school student's mathematical understanding, especially for SELs. My conclusions of these findings extend the current literature, examine the connection between mathematical language and academic language, and highlight the need for a re-examination of ELL classifications.

Extension of Current Literature

The current literature on mathematical discussions mainly focused on high school students, large groups, and ELLs as a single grouping. My study focused on middle school students, the use of small groups and large groups, and broader language classification groups.

I concluded that mathematical discussions can be beneficial for middle school students just as it has been shown to be for high school students, but I highly recommend implementing more structure for mathematical discussions than were detailed in the literature. During my study, I found that it was highly constructive to assign students roles during mathematical discussions. Because my small groups included three students, I implemented three roles: leader, recorder, and gatekeeper. The leader made sure every voice in the group was heard, the recorder took detailed and neat notes to be shared with the large group, and the gatekeeper kept everyone on task and redirected any off-topic behavior. All three students regardless of their job were expected to participate in the discussions equally and to take comprehensive notes. These roles were engaging for students, and I found that they kept the small groups discussions constructive and on-topic (see Appendix D).

I also concluded that small groups were more effective than large groups, although large group discussions were necessary to ensure all students received the same information at the summation of the lesson. In reviewing the audio recordings, I found that during large group discussions, a small percent of students dominated the discussion and that these students tended to be EOs. Small groups allowed all students to participate. Via audio recordings, I was able to hear far more engagement by a variety of students, and although those who dominated the large group discussions also spoke the loudest in their small groups, other voices from their small group were able to collaborate with or challenge them, which they did not feel comfortable doing in the large group. Therefore, I concluded that small group discussion should be considered as equally beneficial to students as large group mathematical discussions.

Furthermore, in reviewing the literature, Chen and Chalhoub-Deville (2016) noted that specific ELL classifications had no significant bearing on their findings. Based on my findings, I did not conclude this to be the case. In my study, I differentiated students as ELLs and SELs. Students who spoke another language at home some, most, or all of the time I classified as ELLs. Students who regularly interacted with adults at home whose first language was not English, but who still communicated with these students predominantly in English, I classified as SELs

(AEMP, 2020). Students who mainly interacted with adults whose first language was English I classified as EOs. In my study, SELs made significantly more growth during the intervention, though all groups showed growth over the twelve weeks. This is evident in my findings illustrated in Figures 1-5 in Chapter IV.

Mathematical Language vs Academic Language

My findings from this study specific to student use of the mathematical register or mathematical language are inconclusive. Some of the measures used showed that students were frequently using mathematical language while others did not, and most of the measures showed a combined use of vernacular language and mathematical language. When I analyzed the mathematical discussions and classwork, students were far more likely to use mathematical language by the end of the intervention. However, they did not use the full scope of the mathematical language during their discussions. Student discussions tended more toward vernacular language usage, especially during small group discussions, rather than the use of Academic English. Written answers conversely showed more academic and mathematical language as evidenced by the later classwork and summative assessment answers, despite drastically less use of vocabulary on later summative assessment answers. Students inconsistent use of the mathematical language may have been impacted by the limited duration of this study or the lack of direct focus on this element of the study as advised by Stephen Krashen and Tracy Terrell’s natural approach to language acquisition (1998). Further and more focused research is needed to clarify these findings.

Re-examining ELL Classifications

As was noted in my findings, SELs showed more growth during this intervention than either of the other subgroups, ELLs and EOs. Why was this subgroup better able to access and grow from the intervention, while the other subgroups were not? Did EOs already have access to certain language skills at home, so they had less room to grow in the first place? Did the ELLs need more direct or longer intervention, or perhaps a different approach to the intervention all together?

The state of California categorizes ELLs in three stages: emerging, expanding, and bridging. These stages are recognized as a continuum and are based on the student’s language proficiency. The Academic English Mastery Program (AEMP), implemented by the Los Angeles Unified School District, offers an expanded approach to classifying students’ English proficiency (AEMP, 2021). They recognized that many of their students would not traditionally be classified as ELLs, as they spoke only English at home. However, these students did not speak Eurocentric Standard English at home and therefore struggled with Eurocentric Academic English at school. The AEMP classified these students as Standard English Learners (SEL) to serve these students more equitably, who were not by current guidelines ELLs, but who still needed to receive support to reach their full academic potential and to address the achievement gap. My findings support the need to address this not traditionally identified group of students.

A student does not exist in isolation. Outside influences heavily impact a student’s readiness to learn and grow in the traditional American classroom (Howard, 2020). Equitable teaching recognizes the influence of race, culture, identity, and the social-emotional joys and challenges that influence the unique needs of each student. However, I believe there is a need for this understanding to include the influences of home language and students who may not have the resource of an adult who is fluent in Standard or Academic English, even if both the student and the adult are fluent in one or more of the many dialects of English. The AEMP strives to address this underserved population, but I believe the research concerning SELs and best teaching practices for SELs need to be implemented on a grander scale, especially in a state as diverse as California.

Limitations

This research project presented several limitations including limited time, a small sample size of student participants, and COVID-19. This study took place over the course of twelve weeks, limiting information that could be gathered and growth that could be measured. Secondly, the sample population was unique to my school site and my particular students. Some of my ethnic populations were very small, being made up of only one or two students. This significantly skewed the percentages of the data for my Black, White, Middle Eastern, and non-Filipino Asian populations. These students were also all considered general education or high-achieving learners. Students with identified or documented special needs were in the small, pullout Math group and did not participate in this study. Finally, this study took place during the COVID-19 pandemic. At the time of the study, the school remained open, but students and teachers were masked at all times. Students were also required to stay home if they had any cold or flu symptoms and to obtain a negative test result before returning to school, which sharply increased the number and duration of student absences and made consistent collection of qualitative data very challenging. The lasting effects of distance learning from the previous year of instruction also had an impact on student learning and, therefore, the findings of this study. In my observation, students in general tended to struggle more with focusing after distance learning. They also seemed more apathetic and quicker to give up when faced with any amount of difficultly. Additionally, students were far more focused on socializing than academics, even for a typical pre-pandemic middle school student. These influences limited the viability of generalizations from the findings of this study.

Additionally, I imposed limitations for this study in two main ways. Firstly, I was both the teacher and the researcher in this study. This duality could affect the validity of the study due to my insight into the supporting research and my known intention of the intervention. Secondly, I am a white cisgender woman, who researched racially and ethnically diverse learners in a suburban school in Northern California. Through this action research project, I wanted to better reach my students to ensure quality and equitable education for all leaners. However, my life experience is one of abundant privilege. Therefore, I must be ever mindful of my power and privilege and do my best to limit its influence in my teaching and interactions with my students.

Summary

The implementation of Common Core Math State Standards has placed a notable emphasis on deeper understanding and critical thinking and requires students to justify and support their mathematical choices and thinking. However, the systemic and grammatical complexities of the mathematical language make mathematical reasoning challenging for all learners but can be especially difficult for ELLs and SELs. (Moschivitch, 2012; AEMP, 2021). In my almost decade of teaching Common Core Math Standards, I have observed the struggle my students have when explaining their thinking, especially at my current school site, where many of my students are ELLs and SELs.

Recent research showed that mathematical discussions were a recognized practice for students of all language levels to actively participate in robust mathematical learning, as well as, providing students ownership in their own learning (Moschkovich, 2012; Sigley and Wilkinson, 2015; Zahner et al., 2012). Informed by this research, the purpose of this action research project was to help deepen my middle school students' mathematical understanding through mathematical discussions, with a focus on my ELLs and SELs. I hoped to expand the recent research in specific relation to SELs as much of the research focuses on ELLs.

The theoretical rational of this research is situated in the larger findings of Lev Vygotsky's zone of proximal development (Vygotsky, 1978) and Stephen Krashen and Tracy Terrell's natural approach to language acquisition (Krashen and Terrell, 1998), which both theorized that learning a new skill occurs through the natural interaction between a learner and a more knowledgeable person. Krashen and Terrell built on Vygotsky's zone of proximal and examine its implications for student language acquisition, specifically ELLs. Both theories underpinned this project to implement mathematical discussions in my classroom to better support ELLs and SELs.

To complete this action research project, data were collected over a twelve-week period. Mathematical discussions occurred once a week in conjunction with regular classroom curriculum. Students engaged in large group discussions as well as small group discussions. Students were grouped in sets of three for these small group discussions: one ELL, one SEL, and one EO. Upon triangulating the data, I concluded that the intervention was successful for all groups but was particularly successful for SELs. As a whole class, students averaged more growth during the study than the twelve weeks prior to the intervention. The student participants during the study averaged 10 more points on their Scaled Score, 4 months more of Grade Equivalent score, 4% higher Percentile Scores, and 2.5% more growth on their Normal Curve Equivalent scores. However, disaggregating the data showed that SELs made the most growth of the three subgroups. SELs on average gained 13% on their Percentile Ranking, 10% on their Normal Curve Equivalent, and 30 points on their internal Scaled Score during this intervention.

This study and its findings were notably limited, i.e., sample size/unique population, my positionality, and COVID-19. These constraints limited the viability of generalizations from the findings of this study.

Based on the data, and supported by the existing literature, my conclusions of these findings were three-fold. Firstly, my conclusions extended the current literature on mathematical discussions. Although most of the related research focused on high school students, large groups, and ELLs as a single grouping, my study focused on middle school students, the use of small groups and large groups, and broader language groups. I concluded that mathematical discussions can be beneficial for middle school students just as it has been shown to be for high school students, but I highly recommend implementing more structure for mathematical discussions than were detailed in the reviewed literature. I also concluded that small group discussion should be considered as equally beneficial to students as large group mathematical discussions to ensure that all students have the opportunity and feel safe to contribute to the conversation. Furthermore, in contrast to the literature, I found that SELs made significantly more growth during the intervention, despite the previous findings that specific ELL classifications had no significant bearing on the correlation of reading scores and math scores (Chen and Chalhoub-Deville, 2016). Secondly, my findings from this study specific to student use of the mathematical register or mathematical language were inconclusive. Some of the measures I used showed that students were frequently using mathematical language while others did not. Further and more focused research is needed to clarify these findings. Lastly, my study highlighted the need for a re-examination of English Language Learners classifications. I believe there is a need to more equitably serve SELs, who are not currently being served in the same way as ELLs, but who still needed to receive support to reach their full academic potential. I believe my findings point to the need for more research concerning SLEs and best teaching practices for teaching these students.

Plan for Future Action

The findings of this study support the conclusion that middle school students' deeper understanding of mathematics is positively influenced by mathematical discussions, particularly for SELs. Based on my findings, I plan to disseminate my findings on three levels: a personal level, a schoolwide level, and a larger community/national level.

On a personal level, I plan to continue using mathematical discussions in my classroom. The related literature and my own findings showed the effectiveness of this intervention when working with diverse language learners. I also found it to be more engaging for my students than extended direct instruction. I also plan to ask more language related questions of my future students. After completing this action research project, I better understand which questions I should be asking to serve my students more equitably. Additionally, I intend to search out more research focused on SELs. Through traditional research and through conversation with my colleagues, I hope to explore this topic further.

At my school site, I plan to present the findings of my student to my peers and to my principal during an upcoming weekly staff meeting. As the lead math teacher, I plan to work with any teacher who is inspired to implement mathematical discussions in their classroom and to support them in this endeavor. I also hope to advise my principal in establishing a regular language survey at our school site, as we currently do not collect this data. With the understanding that this might be a sensitive topic for some families, I hope to guide this process to better serve students who may need extra language support. With this newly acquired data, I hope my school can organize around our resource teachers in supporting best practices for language development. I will also encourage my principal to seek out more professional development opportunities focused on ELL and SEL language acquisition. I believe that my linguistically diverse school community could strongly benefit from this informed focus and that this would help teachers more equitably serve students.

On the community and national level, I believe that mathematical discussions are an important practice in a world of increasing social anxiety and with a political climate that devalues academic reasoning. My students, who experienced a year of remote learning and isolation during one of their most critical social development stages, are struggling to reorient themselves in social situations. Structured discussions are a positive way for students to regain and grow social skills without the interface of technology. Mathematical discussions also help students practice having fact-based and evidence-driven conversations when the media discourse they are exposed to includes neither of these academic gold standards. I also hope that this study is recognized as a call for further research and educational action concerning SELs as a distinct group of language learners. I will certainly be more mindful of these students, and I hope the larger academic community is as well.

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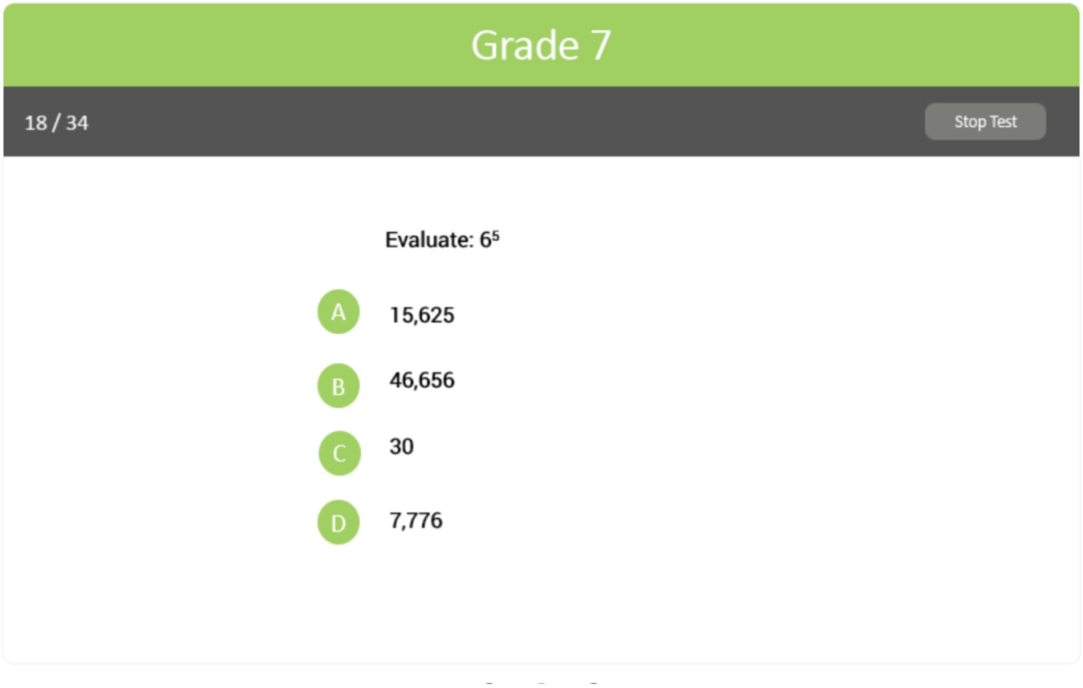
Appendices

Appendix A

Sample STAR Renaissance Assessment Question



Evaluate: 6^5

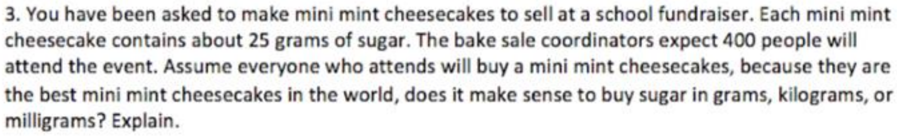
- A 15,625
- B 46,656
- C 30
- D 7,776

(STAR Renaissance, 2022)

Appendix B

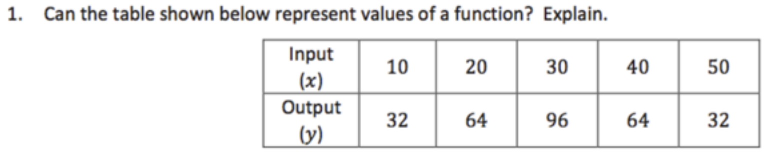
Summative Assessment Questions

8.EE.4



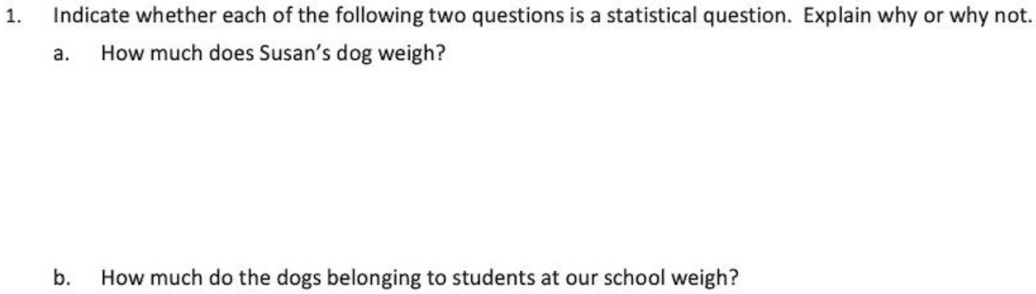
3. You have been asked to make mini mint cheesecakes to sell at a school fundraiser. Each mini mint cheesecake contains about 25 grams of sugar. The bake sale coordinators expect 400 people will attend the event. Assume everyone who attends will buy a mini mint cheesecakes, because they are the best mini mint cheesecakes in the world, does it make sense to buy sugar in grams, kilograms, or milligrams? Explain.

8.F.1



1. Can the table shown below represent values of a function? Explain.

Input (x)	10	20	30	40	50
Output (y)	32	64	96	64	32

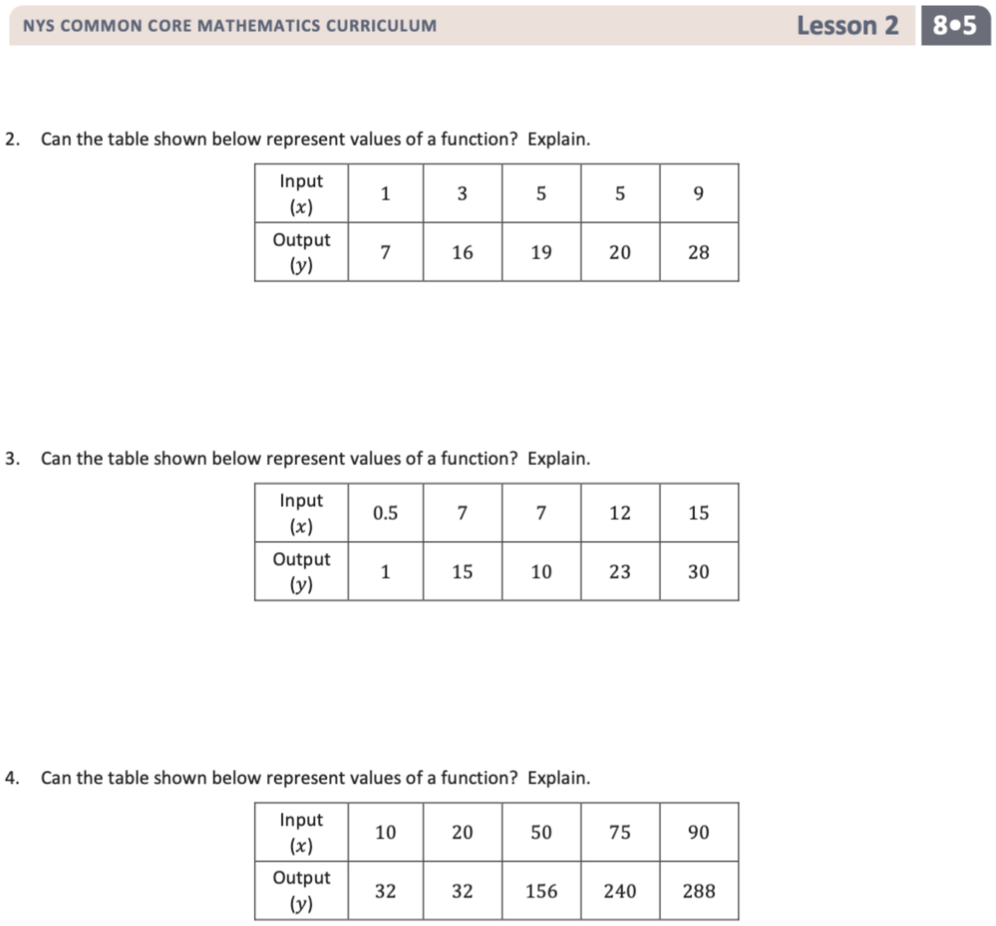


1. Indicate whether each of the following two questions is a statistical question. Explain why or why not.
   a. How much does Susan’s dog weigh?
   b. How much do the dogs belonging to students at our school weigh?

(Common Core, Inc., 2014)

Appendix C

Sample Problems from Engage New York Math



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 8•5

2. Can the table shown below represent values of a function? Explain.

Input (x)	1	3	5	5	9
Output (y)	7	16	19	20	28

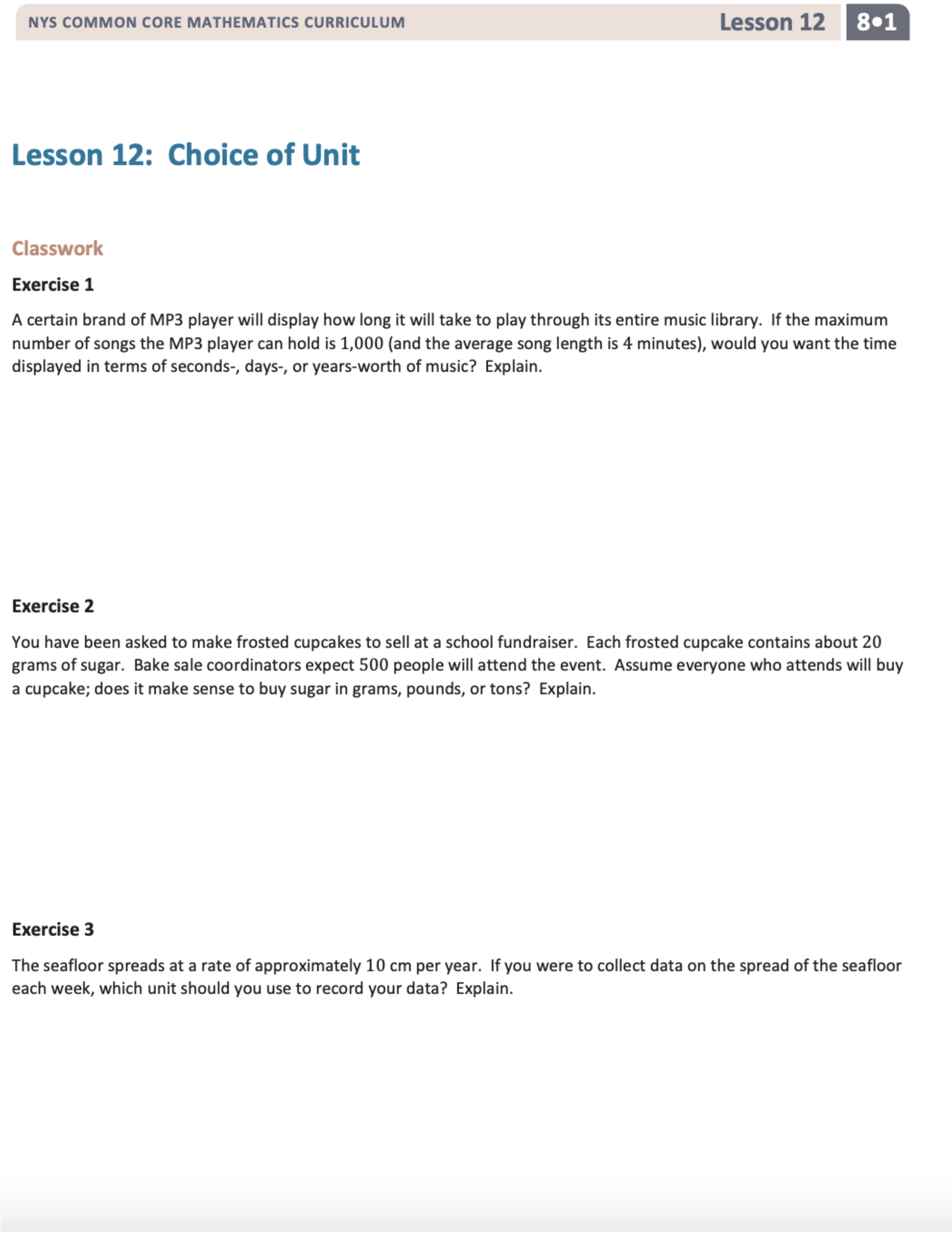
3. Can the table shown below represent values of a function? Explain.

Input (x)	0.5	7	7	12	15
Output (y)	1	15	10	23	30

4. Can the table shown below represent values of a function? Explain.

Input (x)	10	20	50	75	90
Output (y)	32	32	156	240	288

(Common Core, Inc., 2014)



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 12 8•1

Lesson 12: Choice of Unit

Classwork

Exercise 1

A certain brand of MP3 player will display how long it will take to play through its entire music library. If the maximum number of songs the MP3 player can hold is 1,000 (and the average song length is 4 minutes), would you want the time displayed in terms of seconds-, days-, or years-worth of music? Explain.

Exercise 2

You have been asked to make frosted cupcakes to sell at a school fundraiser. Each frosted cupcake contains about 20 grams of sugar. Bake sale coordinators expect 500 people will attend the event. Assume everyone who attends will buy a cupcake; does it make sense to buy sugar in grams, pounds, or tons? Explain.

Exercise 3

The seafloor spreads at a rate of approximately 10 cm per year. If you were to collect data on the spread of the seafloor each week, which unit should you use to record your data? Explain.

(Common Core, Inc., 2014)

Appendix D

Mathematical Discussion Roles

	Description	Helpful Hints
Leader	The leader makes sure that every voice in the group is heard.	You might say… “Samantha, is there anything you would like to add?” “Thank you, Olaf. You make some good points. Does anyone else want to add their thoughts?”
Recorder	The recorder takes the neatest notes, although everyone should be taking their own notes.	Be sure to write using: -full sentences -school appropriate grammar -mathematical language
Gatekeeper	The gatekeeper keeps everyone on task and helps the group stay on-task and on-topic.	You might say… “Let’s bring it back to the question. Elsa, what do you think about number 5?” “Ok, y’all, let’s focus back on the problem, so we have something solid to present to the class.”